6. [15 points] Each of the following concerns a linear, second order, constant coefficient differential equation $y'' + py' + qy = 0$.

    a. [7 points] If the general solution to the problem is $y = c_1 e^{2t} + c_2 e^{4t}$, sketch a phase portrait for the system.

    b. [8 points] Now suppose that for some real-valued α, we have $p = 2\alpha$ and $q = 1$, so that we are considering $y'' + 2\alpha y' + y = 0$. For what values of α, if any

    (i) do all solutions to the differential equation decay to zero?

    (ii) are there solutions that do not decay to zero?

    (iii) will the general solution be a decaying sinusoidal function?