2. [14 points] A model for a population that is susceptible to a disease is the SI (Susceptible, Infected) model. With a few simplifying assumptions, we may model smallpox infections in a population with the SI model

$$S' = -4SI + k(1 - S - I)$$
$$I' = 4SI - I,$$

where S is the fraction of the total population that is susceptible to smallpox and I is the fraction who are infected by the disease. (The remainder of the population is recovered.) We shall consider this with $k = 2$, in which case the equilibrium solutions to the system are $(S, I) = (1, 0)$ and $(S, I) = (1/4, 1/2)$.

a. [5 points] Find the linearization of this system at the critical point $(1, 0)$. Solve the linear system that you obtain.

b. [4 points] Find the linearization of this system at the critical point $(1/4, 1/2)$. Determine the type of critical point this is (that is, whether it is a node, saddle or spiral point, and its stability).