3. [16 points] Consider a model for interacting populations x_1 and x_2 given by

$$x_1' = 2x_1 - \frac{4x_1x_2}{3+x_1}, \quad x_2' = -x_2 + \frac{2x_1x_2}{3+x_1}.$$

a. [2 points] What type of interaction do you think there is between these populations (how does the interaction affect each population)? Explain.

b. [4 points] Find all critical points for this system.

c. [6 points] The Jacobian for this system is $J(x_1, x_2) = \begin{pmatrix} 2 - \frac{12x_2}{(x_1+3)^2} & -\frac{4x_1}{x_1+3} \\ \frac{6x_2}{(x_1+3)^2} & -1 + \frac{2x_1}{x_1+3} \end{pmatrix}$. Classify each of your critical points from (b) by stability and type, and sketch a phase portrait for each. *(This problem part continues on the next page.)*

Problem 3, continued. We are considering the system

$$x_1' = 2x_1 - \frac{4x_1x_2}{3+x_1}, \quad x_2' = -x_2 + \frac{2x_1x_2}{3+x_1}.$$

c. *Continued: we are solving the problem*

The Jacobian for this system is $J(x_1, x_2) = \begin{pmatrix} 2 - \frac{12x_2}{(x_1+3)^2} & -\frac{4x_1}{x_1+3} \\ \frac{6x_2}{(x_1+3)^2} & -1 + \frac{2x_1}{x_1+3} \end{pmatrix}$. Classify each of your critical points from (b) by stability and type, and sketch a phase portrait for each.

d. [4 points] If the population of x_1 was initially large and that of x_2 small, sketch a qualitatively accurate graph of x_1 and x_2 as functions of time. What happens to the populations for large times?