

6. [12 points] For the following, identify each as true or false by circling “True” or “False” as appropriate. Then, if it is true, provide a short (one sentence) explanation indicating why it is true; if false, explain why or provide a counter-example.

a. [3 points] Let  $\mathbf{A}$  be a  $3 \times 3$  matrix with characteristic polynomial  $p(\lambda) = \lambda^3 + 4\lambda^2 + \lambda - 6$ . Then the origin is an asymptotically stable critical point of the system  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ .

True                      False

b. [3 points] Consider the equation  $y' = f(t, y)$ , with  $f$  continuous for all values of  $t$  and  $y$ . We can solve this either by using an integrating factor or by separating variables (though in the latter case we may not be able to get an explicit solution for  $y$ ).

True                      False

c. [3 points] While we cannot solve the nonlinear system  $x' = x - x^2 - xy + \sin(t)$ ,  $y' = y + xy$ , we can obtain a good qualitative understanding of solutions by linearizing around critical points and sketching a phase portrait.

True                      False

d. [3 points] Long-term solutions to the system  $y'' + 4y = 3 \cos(4t)$  will be periodic.

True                      False