6. [12 points] For the following, identify each as true or false by circling “True” or “False” as appropriate. Then, if it is true, provide a short (one sentence) explanation indicating why it is true; if false, explain why or provide a counter-example.

a. [3 points] Let $A$ be a $3 \times 3$ matrix with characteristic polynomial $p(\lambda) = \lambda^3 + 4\lambda^2 + \lambda - 6$. Then the origin is an asymptotically stable critical point of the system $x' = Ax$.

True False

b. [3 points] Consider the equation $y' = f(t, y)$, with $f$ continuous for all values of $t$ and $y$. We can solve this either by using an integrating factor or by separating variables (though in the latter case we may not be able to get an explicit solution for $y$).

True False

c. [3 points] While we cannot solve the nonlinear system $x' = x - x^2 - xy + \sin(t), y' = y + xy$, we can obtain a good qualitative understanding of solutions by linearizing around critical points and sketching a phase portrait.

True False

d. [3 points] Long-term solutions to the system $y'' + 4y = 3\cos(4t)$ will be periodic.

True False