6. [12 points] For the following, identify each as true or false by circling "True" or "False" as appropriate. Then, if it is true, provide a short (one sentence) explanation indicating why it is true; if false, explain why or provide a counter-example.

a. [3 points] Let $\mathbf{A}$ be a 3×3 matrix with characteristic polynomial $p(\lambda) = \lambda^3 + 4\lambda^2 + \lambda - 6$. Then the origin is an asymptotically stable critical point of the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

True **[False]**

Solution: We note that $p(1) = 1 + 4 + 1 - 6 = 0$, so $\lambda = 1$ is an eigenvalue. Thus there is a solution $\mathbf{x} = \mathbf{v}e^t$ to the system, and it is clearly unstable.

b. [3 points] Consider the equation $y' = f(t, y)$, with f continuous for all values of t and y. We can solve this either by using an integrating factor or by separating variables (though in the latter case we may not be able to get an explicit solution for y).

True **[False]**

Solution: This is false. Consider $y' = y^2 + x$. This is neither linear (so integrating factors do not work) nor separable.

c. [3 points] While we cannot solve the nonlinear system $x' = x - x^2 - xy + \sin(t)$, $y' = y + xy$, we can obtain a good qualitative understanding of solutions by linearizing around critical points and sketching a phase portrait.

True **[False]**

Solution: Because this is not autonomous we will be unable to find equilibrium solutions (that is, critical points), and the phase portrait will depend on time.

d. [3 points] Long-term solutions to the system $y'' + 4y = 3\cos(4t)$ will be periodic.

[True] False

Solution: The solution to this will be $y_c = C\cos(2t - \delta_1) + R\cos(4t - \delta_2)$, which will be periodic with period π.