3. [12 points] For parts (a) and (b), identify each as true or false, and give a short mathematical calculation, with explanation, justifying your answer. For part (c) the statement is false. Explain why.

a. [4 points] If L is a linear second-order differential operator, y_1 and y_2 are non-zero functions for which $L[y_1] = L[y_2] = 0$, and y_3 is a function for which $L[y_3] = \frac{3}{2+t^2}$, then for any c_1, c_2, and c_3, $y = c_1y_1 + c_2y_2 + c_3y_3$ solves $L[y] = \frac{3}{2+t^2}$.

True False

b. [4 points] If L is a linear second-order differential operator with continuous coefficients, and y_1 and y_2 are non-zero functions satisfying $L[y] = 0$, $y_1(0) = y_2'(0) = 0$ and $y_1'(0) = y_2(0) = 1$, then a general solution to $L[y] = 0$ is given by $y = c_1y_1 + c_2y_2$.

True False

c. [4 points] Suppose **A** is a real-valued 3×3 matrix, and that the three curves shown to the right are the component plots of a solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$, as indicated. Explain why the statement "eigenvalues of **A** must be complex-valued" is false.