3. [12 points] For parts (a) and (b), identify each as true or false, and give a short mathematical calculation, with explanation, justifying your answer. For part (c) the statement is false. Explain why.

a. [4 points] If \( L \) is a linear second-order differential operator, \( y_1 \) and \( y_2 \) are non-zero functions for which \( L[y_1] = L[y_2] = 0 \), and \( y_3 \) is a function for which \( L[y_3] = \frac{3}{2 + t^2} \), then for any \( c_1, c_2, \) and \( c_3 \), \( y = c_1 y_1 + c_2 y_2 + c_3 y_3 \) solves \( L[y] = \frac{3}{2 + t^2} \).

True False

b. [4 points] If \( L \) is a linear second-order differential operator with continuous coefficients, and \( y_1 \) and \( y_2 \) are non-zero functions satisfying \( L[y] = 0 \), \( y_1(0) = y_2(0) = 0 \) and \( y_1'(0) = y_2(0) = 1 \), then a general solution to \( L[y] = 0 \) is given by \( y = c_1 y_1 + c_2 y_2 \).

True False

c. [4 points] Suppose \( A \) is a real-valued \( 3 \times 3 \) matrix, and that the three curves shown to the right are the component plots of a solution to \( x' = Ax \), as indicated. Explain why the statement “eigenvalues of \( A \) must be complex-valued” is false.