

8. [12 points] Suppose that for some nonlinear second-order differential equation $y'' = f(y)$ we can write an equivalent system of two first-order differential equations $x_1' = F(x_1, x_2)$, $x_2' = G(x_1, x_2)$. Critical points of the latter are $\mathbf{x}_0 = (0, 0)$ and $\mathbf{x}_1 = (1, 0)$. The Jacobian at these points is $\mathbf{J}(\mathbf{x}_0) = \begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix}$ and $\mathbf{J}(\mathbf{x}_1) = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix}$.

a. [8 points] Sketch a phase portrait for the nonlinear system.

b. [4 points] Based on your phase portrait, sketch a qualitatively accurate graph of y as a function of t if we start with the initial condition $y(0) = 0$, $y'(0) = 1$.