3. [12 points] For parts (a) and (b), identify each as true or false, and give a short mathematical calculation, with explanation, justifying your answer. For part (c) the statement is false. Explain why.

a. [4 points] If L is a linear second-order differential operator, y_1 and y_2 are non-zero functions for which $L[y_1] = L[y_2] = 0$, and y_3 is a function for which $L[y_3] = \frac{3}{2+t^2}$, then for any c_1, c_2, and c_3, $y = c_1 y_1 + c_2 y_2 + c_3 y_3$ solves $L[y] = \frac{3}{2+t^2}$.

True **[False]**

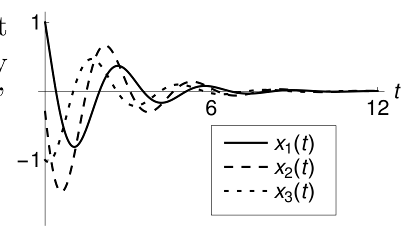
Solution: Because L is linear, we have $L[c_1 y_1 + c_2 y_2 + c_3 y_3] = c_1 L[y_1] + c_2 L[y_2] + c_3 L[y_3] = (c_1 + c_2)(0) + c_3(\frac{3}{2+t^2}) \neq \frac{3}{2+t^2}$ (unless $c_3 = 1$).

b. [4 points] If L is a linear second-order differential operator with continuous coefficients, and y_1 and y_2 are non-zero functions satisfying $L[y] = 0$, $y_1(0) = y_2'(0) = 0$ and $y_1'(0) = y_2(0) = 1$, then a general solution to $L[y] = 0$ is given by $y = c_1 y_1 + c_2 y_2$.

[True] False

Solution: Note that the Wronskian of y_1 and y_2, at $t = 0$, is $W[y_1, y_2](0) = y_1(0)y_2'(0) - y_1'(0)y_2(0) = -1 \neq 0$. Thus y_1 and y_2 are linearly independent at zero (and hence everywhere), and a general solution is given by $y = c_1 y_1 + c_2 y_2$.

c. [4 points] Suppose $\mathbf{A}$ is a real-valued 3×3 matrix, and that the three curves shown to the right are the component plots of a solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$, as indicated. Explain why the statement "eigenvalues of $\mathbf{A}$ must be complex-valued" is false.



Solution: If $\mathbf{A}$ is a real-valued 3×3 matrix, complex-valued eigenvalues must come in complex-conjugate pairs, and there are three eigenvalues. Here we see decaying oscillatory behavior, so there must be a (pair of complex-conjugate) eigenvalue(s), and one real one.