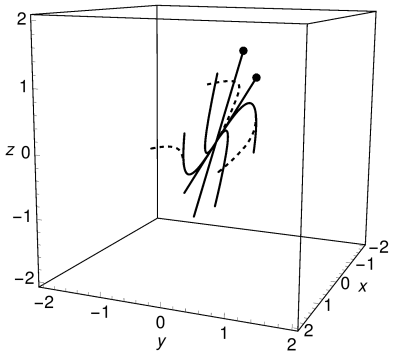
2. [10 points] In this problem we consider a linearization of the Lorenz system that we considered in lab 5, with $\eta = \sqrt{8(r-1)/3}$,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}' = \begin{pmatrix} -10 & 10 & 0 \\ 1 & -1 & -\eta \\ \eta & \eta & -2.67 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

a. [5 points] For some value of r, if phase portrait is shown in the figure to the right, below, what can you say about the eigenvalues and eigenvectors of this system (please, whatever you do, do not try to calculate exact values from the system)? The solid black trajectories lie in a plane. The dashed trajectories start to the left the plane as you look at it. The two black points are, approximately, $(1, 0.75, 1.5)$ and $(1, 1, 1.25)$. You should be able to specify at least two eigenvectors and the relative values of the eigenvalues.



b. [5 points] For a different value of r, the general solution to the system is, approximately,

$$\begin{aligned} x &= c_1 e^{-12t} + c_2\,(0.5\,\cos(0.4\,t) - 0.05\,\sin(0.4\,t))\,e^{-t} + \\ &\qquad c_3\,(0.05\,\cos(0.4\,t) + 0.5\,\sin(0.4\,t))\,e^{-t} \\ y &= -0.4\,c_1\,e^{-12t} + c_2\,(0.5\,\cos(0.4\,t) - 0.05\,\sin(0.4\,t))\,e^{-t} + \\ &\qquad c_3\,(0.05\,\cos(0.4\,t) + 0.5\,\sin(0.4\,t))\,e^{-t} \\ z &= -0.1\,c_1\,e^{-12t} + 0.75\,c_2\,e^{-t}\cos(0.4\,t) + 0.75\,c_3\,e^{-t}\sin(0.4\,t). \end{aligned}$$

What are the eigenvalues and eigenvectors of the coefficient matrix for the linear system in this case?