4. [12 points] Consider the system of differential equations given by $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where $\mathbf{A}$ is a real-valued 2×2 matrix and $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.

a. [6 points] Suppose that the eigenvalues and eigenvectors of $\mathbf{A}$ are $\lambda = -1 \pm i$, with $\mathbf{v} = \begin{pmatrix} 2 \pm i \\ 1 \end{pmatrix}$. If $\mathbf{x}$ solves $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, sketch the trajectory for $\mathbf{x}$ in the phase plane.

b. [6 points] Suppose that eigenvalues and eigenvectors of $\mathbf{A}$ are $\lambda_1 = 1$ and $\lambda_2 = 2$, with $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$. If $\mathbf{x}(0) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$, as $t \to \infty$, which of the following is most correct, and why? (i) $x_2 \approx 2x_1$; (ii) $x_2 \approx -\frac{1}{2}x_1$; (iii) $x_2 \approx -\frac{1}{2}x_1 - 1$; (iv) $x_2 \approx -\frac{1}{2}x_1 - k$, with $k > 1$.