5. [12 points] In lab 6 we considered the Fitzhugh-Nagumo model for the behavior of a neuron,

$$v' = v - \frac{1}{3}v^3 - w + I_{ext}, \qquad \tau w' = v + a - bw.$$

In this problem we analyze this with the parameters $\tau = 1$, $a = \frac{1}{3}$, and $b = 1$.

a. [3 points] Find the v- and w-nullclines, and show that there is a single critical point (v_c, w_c) in this case. Find the critical point in terms of the externally applied voltage I_{ext}.

b. [3 points] Linearize the system at the critical point (write your linearization in terms of v_c and w_c—do not plug in the values you found for v_c and w_c). How is the solution to your linearized system related to the solution of the original nonlinear system?

c. [6 points] Show that the critical point is in this case is always stable. Determine any values of v_c or w_c at which the behavior at the critical point changes. Explain how this result is different from that which you saw in lab.

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