

6. [12 points] Consider the nonlinear system

$$x' = 3x - y - x^2, \quad y' = -\alpha + x - y,$$

where α is a real-valued parameter.

a. [4 points] Find all critical points for the system, and show that for $\alpha > -1$ there are two critical points, if $\alpha = -1$ there is one, and if $\alpha < -1$ there are none.

b. [8 points] Let $\alpha = 0$: then the system has two critical points, $(0, 0)$ and $(2, 2)$. Sketch a phase portrait for the nonlinear system by linearizing at critical points and determining the resulting behavior in the phase plane.