6. [12 points] For each of the following, circle True or False to indicate whether the statement is true or not, and provide a one-sentence explanation. Note that without an explanation no credit will be awarded.
a. [3 points] Given any two solutions $y_{1}$ and $y_{2}$ to an equation $y^{\prime \prime}+p(x) y^{\prime}+q(x) y=0$ (where $p(x)$ and $q(x)$ are continuous), any other solution to the equation may be written as $y=c_{1} y_{1}+c_{2} y_{2}$ for some constants $c_{1}$ and $c_{2}$.

True False
b. [3 points] $\mathfrak{L}^{-1}\left\{\frac{3 s-6}{s^{2}+4 s+20}\right\}=3 e^{-2 t} \cos (4 t)-\frac{3}{2} e^{-2 t} \sin (4 t)$.

True False
c. [3 points] Any 2nd or higher order system of ordinary differential equations $y^{(n)}=$ $f\left(t, y, y^{\prime}, \ldots, y^{(n-1)}\right)(n \geq 2)$ may be written as a system of first-order ordinary differential equations.

True
False
d. [3 points] Suppose we approximate the solution to $y^{\prime}=f(x, y)$ on the domain $0 \leq x \leq 10$ using a numerical method. With $h=.1$ we get $y(10) \approx 1.501$; with $h=.01, y(10) \approx 1.487$; and with $h=0.001$ or $h=0.0001, y(10) \approx 1.486$. Thus the exact value of $y(10)$ is to three decimal places 1.486, the errors when $h=0.1$ and $h=0.01$ are 0.015 and 0.001 respectively, and it is most likely that the numerical method used was the improved Euler method.

True
False

