7. [14 points] The van der Pohl oscillator is a circuit that may be modeled with the system of differential equations

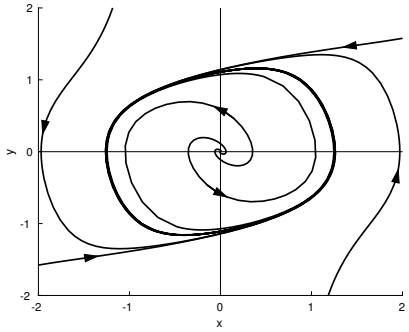
$$x' = -y, \qquad y' = x + (a - y^2)\, y,$$

where x is the charge on a capacitor in the circuit and y is current in the circuit, scaled appropriately. The constant a is a parameter in the system.

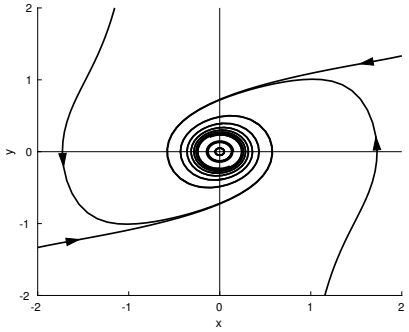
a. [3 points] Find all critical points for this system.

b. [6 points] The two phase portraits (I and II) shown below are generated for the system two of the three cases $a = -1$, $a = 0$ or $a = 1$. By doing a linear analysis of the system at your critical points, determine which cases these match and explain why.

I.



II.



c. [5 points] Based on your linear analysis, sketch a phase portrait for the last of the three cases $a = -1$, $a = 0$, or $a = 1$.