6. [12 points] For each of the following, circle True or False to indicate whether the statement is true or not, and provide a one-sentence explanation. ***Note** that without an explanation no credit will be awarded.*

a. [3 points] Given any two solutions y_1 and y_2 to an equation $y'' + p(x)y' + q(x)y = 0$ (where $p(x)$ and $q(x)$ are continuous), any other solution to the equation may be written as $y = c_1y_1 + c_2y_2$ for some constants c_1 and c_2.

True $\boxed{\text{False}}$

Solution: This is false; if the solutions y_1 and y_2 are not linearly independent their linear combination does not give the general solution to the differential equation.

b. [3 points] $\mathfrak{L}^{-1}\{\frac{3s-6}{s^2+4s+20}\} = 3e^{-2t}\cos(4t) - \frac{3}{2}e^{-2t}\sin(4t)$.

True $\boxed{\text{False}}$

Solution: Note that $\frac{3s-6}{s^2+4s+20} = \frac{3(s+2)-12}{(s+2)^2+16}$, so that the inverse transform should be $3e^{-2t}\cos(4t) - 3e^{-2t}\sin(4t)$.

c. [3 points] Any 2nd or higher order system of ordinary differential equations $y^{(n)} = f(t, y, y', \dots, y^{(n-1)})$ $(n \geq 2)$ may be written as a system of first-order ordinary differential equations.

$\boxed{\text{True}}$ False

Solution: Let $x_1 = y$, $x_2 = y'$,..., $x_n = y^{(n-1)}$, to get the system $x_1' = x_2$, $x_2' = x_3$,..., $x_n' = f(t, x_1, x_2, \dots, x_n)$.

d. [3 points] Suppose we approximate the solution to $y' = f(x, y)$ on the domain $0 \leq x \leq 10$ using a numerical method. With $h = .1$ we get $y(10) \approx 1.501$; with $h = .01$, $y(10) \approx 1.487$; and with $h = 0.001$ or $h = 0.0001$, $y(10) \approx 1.486$. Thus the exact value of $y(10)$ is to three decimal places 1.486, the errors when $h = 0.1$ and $h = 0.01$ are 0.015 and 0.001 respectively, and it is most likely that the numerical method used was the improved Euler method.

True $\boxed{\text{False}}$

Solution: All of the statements are reasonable except for the assertion that improved Euler was used. We would expect that the error would drop by a factor of $10^2 = 100$ (that is, that the error for $h = 0.01$ would be on the order of 0.00015, not 0.001).