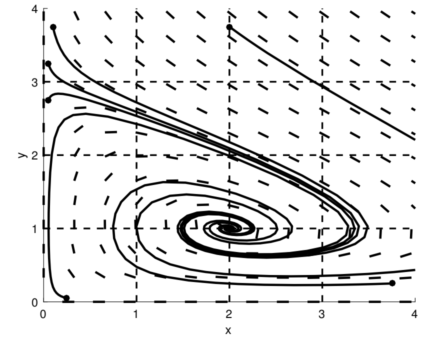
**8**. [12 points] Consider the system

$$x' = ax + 2xy$$
$$y' = 3y - y^2 + bxy$$

where $a$ and $b$ are constants. The direction field and phase portrait for the system are shown in the figure to the right. (Dots indicate initial conditions for the trajectories shown.)




**a**. [6 points] What are $a$ and $b$? Be sure to explain your answer.

*Solution:* From the phase portrait, it appears that there are three critical points, at $(0,0)$, $(0,3)$ and $(2,1)$. The given system will have critical points when

$$0 = x(a + 2y) \qquad \text{and}$$
$$0 = y(3 - y + bx).$$

Thus, from the first equation we have $x = 0$ or $y = -a/2$. If $x = 0$ the second equation requires that $y = 0$ or $y = 3$, which give the first two critical points. Thus $a = -2$ so that $y = 1$ for the third critical point. If $y = 1$ we have $2 + bx = 0$, and we know that $x = 2$. Thus $b = -1$.

**b**. [6 points] Suppose that $x$ and $y$ are populations of interacting species. What type of interaction is being modeled here? Explain what the phase portrait shown tells you about the behavior and expected long-term values of the populations and sketch a representative solution ($x$ and $y$) against $t$.

*Solution:* With $a = -2$ and $b = -1$, we have the system

$$x' = -2x + 2xy$$
$$y' = 3y - y^2 - xy.$$

Thus, population $x$ is helped by the interaction while population $y$ is hindered; we might guess that this is a predator/prey relationship with $x$ as the predator and $y$ as the prey. From the phase portrait, we see that if we start with $y = 0$ (no prey), $x \to 0$; and if $x = 0$ (no predators), $y \to 3$ (the environmental limiting value). If $x > 0$ and $y > 0$, we expect to see the populations of $x$ and $y$ oscillating around $x = 2$, $y = 1$ and as $t \to \infty$ that we will see $x \to 2$ and $y \to 1$. Thus our sketch of solutions is the following: