6. [10 points] If we make a small typographical error when writing out the Lorenz system that we studied in lab 5, we obtain the system

$$x' = \sigma(-x + y)$$
$$y' = ry - x - xz$$
$$z' = -bz + xy$$

a. [5 points] As with the Lorenz system, one critical point of this system is $(0, 0, 0)$. Find a linear system that approximates the system near $(0, 0, 0)$.

b. [5 points] If $b = 5$, $\sigma = 1$, and $r = 1/4$, the eigenvalues and eigenvectors of the coefficient matrix of the linearized system you found in (a) are approximately $\lambda_1 = -5$ and $\lambda_{2,3} = -\frac{3}{8} \pm \frac{7}{9}\, i$, with $\mathbf{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, and $\mathbf{v}_{2,3} = \begin{pmatrix} \frac{5}{8} \pm \frac{6}{7}\, i \\ 1 \\ 0 \end{pmatrix}$. Describe phase space trajectories in this case. If we start with an initial condition $(x, y, z) = (0.5, 0.5, 0)$, sketch the trajectory in the phase space.