7. [15 points] Consider a mass-spring system with a nonlinear "soft" spring, for which the displacement x of a mass attached to the spring is modeled by

$$x'' + 2\gamma_0 x' + k(x - x^2) = 0.$$

a. [4 points] Rewrite this as a system in $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x' \end{pmatrix}$.

b. [5 points] Find all critical points for your system from (a).

Problem 7, continued.
We are solving

$$x'' + 2\gamma_0 x' + k(x - x^2) = 0.$$

You may want to write your system from (a) here:

c. [6 points] Let $\gamma_0 = 4$ and $k = 18$. Sketch the phase plane for the system in this case by linearizing about all critical points and determining local behavior. Using your sketch, what do you expect to happen to a solution that starts with the initial condition $x(0) = 0.8$, $x'(0) = y(0) = 0.2$? *(Note: for this part of the problem you should assume that the original equation is in fact well-defined for $x < 0$.)*