8. [15 points] For each of the following, identify the statement as true or false by circling "True" or "False" as appropriate, and provide a short (one or two sentence) explanation indicating why that answer is correct.

a. [3 points] Two linearly independent solutions of $x'' + 6x' + 9x = 0$ are $x_1 = e^{-3t}$ and $x_2 = te^{-3t}$. Thus two linearly independent solutions of $\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -9 & -6 \end{pmatrix} \mathbf{x}$ are $\mathbf{x}_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{-3t}$ and $\mathbf{x}_2 = \begin{pmatrix} 1 \\ -3 \end{pmatrix} te^{-3t}$.

True False

b. [3 points] If $\mathbf{A}$ is a real-valued 5×5 matrix with 5 distinct eigenvalues, not necessarily real, and if the real parts of all of the eigenvalues are negative, then $\mathbf{x} = \mathbf{0}$ is an asymptotically stable critical point of $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

True False

c. [3 points] If the nonlinear system $\mathbf{x}' = \mathbf{f}(\mathbf{x})$ has an unstable isolated critical point $\mathbf{x} = \mathbf{x}_0$, then any solution to the system will eventually get infinitely far from $\mathbf{x}_0$.

True False

d. [3 points] Suppose that the nonlinear system $x' = F(x, y)$, $y' = G(x, y)$ has an isolated critical point $(x, y) = (1, 2)$. If we are able to linearize the system at this critical point and the eigenvalues of the resulting coefficient matrix are real-valued and non-zero, we can deduce the stability of the critical point from the linearization.

True False

e. [3 points] $\mathcal{L}^{-1}\{\frac{e^{-2s}s}{(s+1)^2+4}\} = e^{-(t-2)} \cos(2(t-2))u_2(t)$

True False