7. [16 points] Our model for a ruby laser is, with $N =$ the population inversion of atoms and $P =$ the intensity of the laser,

$$N' = \gamma A - \gamma N(1 + P), \quad P' = P(N - 1).$$

In lab we found that the critical points of this system are $(N, P) = (A, 0)$ and $(N, P) = (1, A - 1)$. For this problem we will assume that $\gamma = \frac{1}{2}$; A is, of course, also a constant.

a. [4 points] Find a linear system that approximates the nonlinear system near the critical point $(A, 0)$. Show that if $A < 1$ this critical point is asymptotically stable, and if $A > 1$ it is unstable.

b. [6 points] Suppose that the linear system you obtained in (a) is, for some value of A, $u' = -\frac{1}{2}\,u - v$, $v' = v$. Sketch a phase portrait that shows solution trajectories of the linear system. Explain how these trajectories are related to trajectories in the (N, P) phase plane.

Problem 7, cont. We are considering the system

$$N' = \frac{1}{2}A - \frac{1}{2}N(1+P), \quad P' = P(N-1),$$

which has critical points $(N,P) = (A,0)$ *and* $(N,P) = (1, A-1)$.

c. [6 points] Suppose that, for the value of A used in (b), the coefficient matrix for the linear system approximating (N,P) near the critical point $(1, A-1)$ is $\begin{pmatrix} -1 & -\frac{1}{2} \\ 1 & 0 \end{pmatrix}$, which has eigenvalues $\lambda = \frac{1}{2}(-1 \pm i)$. Using this information with your work in (b), sketch a representative solution curve for P as a function of t, if $P(0) = 0.01$ when $N(0) = 0$.