

3. [12 points] Suppose a model for a physical system (e.g., a circuit or a mass-spring system) is given by the differential equation  $L[y] = y'' + ay' + by = k$  (where  $a$ ,  $b$ , and  $k$  are real numbers).
- a. [4 points] If the solution to the problem with some initial conditions is  $y = e^{-t} \cos(2t) - e^{-t} \sin(2t) + 2$ , what can you say about  $a$ ,  $b$ , and  $k$ ?
- b. [4 points] If the solution to the problem with some initial conditions is  $y = e^{-t} \cos(2t) - e^{-t} \sin(2t) + 2$ , sketch a phase portrait for the system. Be sure it is clear how you obtain your solution.
- c. [4 points] Now suppose that the solution to the problem with some initial conditions is  $y = e^{-t} \cos(2t) - e^{-t} \sin(2t) + 2$ , and that at some time  $t = t_0$  we remove the forcing term ( $k$ ). Write a single differential equation you could solve to find  $y$  for all  $t \geq 0$ . What initial conditions apply at  $t = 0$ ?