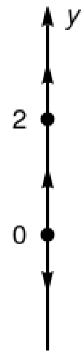
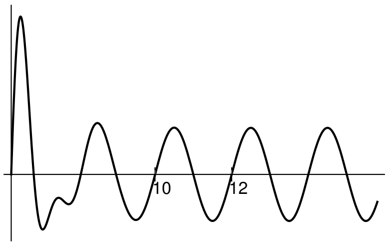
2. [12 points] For each of the following, give an example, as indicated.

a. [4 points] Give a first-order differential equation that could have the phase line shown to the right.



Solution: We see that there are critical points at $y = 0$ and $y = 2$, and that the first is unstable and the second semistable. This suggests the equation $y' = y(y-2)^2$. Checking, we see that for $y < 0$, $y' < 0$, and $y > 0$ gives $y' > 0$ (for $y \neq 2$), so this works.

b. [4 points] Give a second-order, linear, constant-coefficient, nonhomogeneous differential equation that could have the response shown to the right.



Solution: We note this has a likely decaying oscillatory transient, and a steady sinusoidal response. Thus we expect the equation is something like $y'' + y' + 2y = \cos(\omega t)$. We note that the period of the steady response is 2, so $\omega = \pi$.

c. [4 points] Give a system of two linear, first-order, constant-coefficient differential equations which have an isolated critical point at the origin that is an unstable saddle point.

Solution: A system such as $\mathbf{x}' = \mathbf{A}\mathbf{x} = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \mathbf{x}$ will have the desired characteristics. Eigenvalues are $\lambda = \pm 1$, and because $\det(\mathbf{A}) \neq 0$ the origin is the only critical point and is thus isolated.