Math 115 — Practice for Exam 1

Generated October 6, 2019

Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

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1. This exam has 18 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2017	1	7	cats	10	
Fall 2014	1	5	caffeine	11	
Fall 2008	1	5	Lehman	12	
Fall 2017	1	2	globe	13	
Fall 2009	1	3	baby names	12	
Winter 2018	1	7	orchard robots	10	
Fall 2015	1	6		6	
Fall 2014	1	1	AA Detroit	10	
Fall 2008	1	9	Golden Gate	8	
Fall 2017	1	10		6	
Fall 2010	1	8	apples	18	
Winter 2017	1	10		4	
Fall 2018	1	8		10	
Fall 2018	1	1	unicycle	11	
Winter 2019	1	5		15	
Fall 2017	1	4		14	
Fall 2013	1	9		10	
Fall 2009	1	9		8	
Total				188	

Recommended time (based on points): 169 minutes

7. [10 points] Two housecats, Jasper and Zander, escape from their house at the same time and travel along a straight line between their house and a tree. Let $J(t)$ (respectively $Z(t)$) be Jasper's (respectively Zander's) distance, in feet, from the tree t seconds after escaping. The table below shows some of the values of $J(t)$ and $Z(t)$. Assume that $J(t)$ is invertible.

t	6	17	22	31	37
$J(t)$	41	33	21	14	2
$Z(t)$	39	32	31	36	43

a. [2 points] What is Jasper's average velocity for $6 \le t \le 22$? *Be sure to include units.*

Answer: $\frac{21-41}{22-6} = -\frac{20}{16} = -\frac{5}{4} = -1.25$ **ft/sec.**

b. [2 points] Estimate $Z'(31)$. *Remember to show your work.*

You can approximate it using either:

- Average rate of change in $[31, 37]$: $\dfrac{43-36}{37-31} = 7/6$.
- Average rate of change in $[22, 31]$: $\dfrac{36-31}{31-22} = 5/9$.
- Averaging these two: $\frac{1}{2}(7/6 + 5/9) = 0.86\overline{1}$, or
- Average rate of change in $[22, 37]$: $\dfrac{43-31}{37-33} = 0.8$.

Answer: (picking one) 7/6

c. [3 points] Circle the one statement below that is best supported by the equation

$$Z(J^{-1}(8) - 4) = 34.$$

i. 34 seconds after escaping, Zander is 4 feet closer to the tree than Jasper was 8 seconds after escaping.

ii. $\boxed{\text{Four seconds before Jasper is 8 feet from the tree, Zander is 34 feet from the tree.}}$

iii. When Jasper is 4 feet further from the tree than he was 8 seconds after escaping, Zander is 34 feet from the tree.

iv. When Jasper is 4 feet closer to the tree than he was 8 seconds after escaping, Zander is 34 feet from the tree.

v. Four seconds after Jasper is 8 feet from the tree, Zander is 34 feet from the tree.

d. [3 points] Circle the one statement below that is best supported by the equation

$$(J^{-1})'(3) = -0.2.$$

i. In the third second after leaving the house, Jasper travels about 0.2 feet.

ii. When Jasper is 3 feet from the tree, he is traveling about 0.2 feet/second slower than he was one foot earlier.

iii. Jasper gets about 1.5 feet closer to the tree during the third second after leaving the house.

iv. $\boxed{\text{It takes Jasper about one-tenth of a second to go from 3 feet to 2.5 feet from the tree.}}$

v. One-half of a second before Jasper was 3 feet from the tree, he was about 2.9 feet from the tree.

5. [11 points] Oren, a Math 115 student, realizes that the more caffeine he consumes, the faster he completes his online homework assignments. Before starting tonight's assignment, he buys a cup of coffee containing a total of 100 milligrams of caffeine.

Let $T(c)$ be the number of minutes it will take Oren to complete tonight's assignment if he consumes c milligrams of caffeine. Suppose that T is continuous and differentiable.

a. [2 points] Circle the ONE sentence below that is best supported by the statement "the more caffeine he consumes, the faster he completes his online homework assignments."

i. $T'(c) \geq 0$ for every value c in the domain of T.

ii. $\boxed{T'(c) \leq 0 \text{ for every value } c \text{ in the domain of } T.}$

iii. $T'(c) = 0$ for every value c in the domain of T.

b. [1 point] Explain, in the context of this problem, why it is reasonable to assume that $T(c)$ is invertible.

Solution: Since the more caffeine Oren consumes the faster he is able to finish his homework, $T(c)$ is a decreasing function. Thus, $T(c)$ is invertible.

c. [2 points] Interpret the equation $T^{-1}(100) = 45$ in the context of this problem. Use a complete sentence and include units.

Solution: In order for Oren to complete his homework assignment in 100 minutes, he must consume 45 milligrams of caffeine.

d. [3 points] Suppose that p and k are constants. In the equation $T'(p) = k$, what are the units on p and k?

Answer: Units on p are **milligrams of caffeine**

Answer: Units on k are **minutes per milligram of caffeine**

e. [3 points] Which of the statements below is best supported by the equation $(T^{-1})'(20) = -10$? *Circle the* ONE *best answer.*

i. If Oren has consumed 20 milligrams of caffeine, then consuming an additional milligram of caffeine will save him about 10 minutes on tonight's assignment.

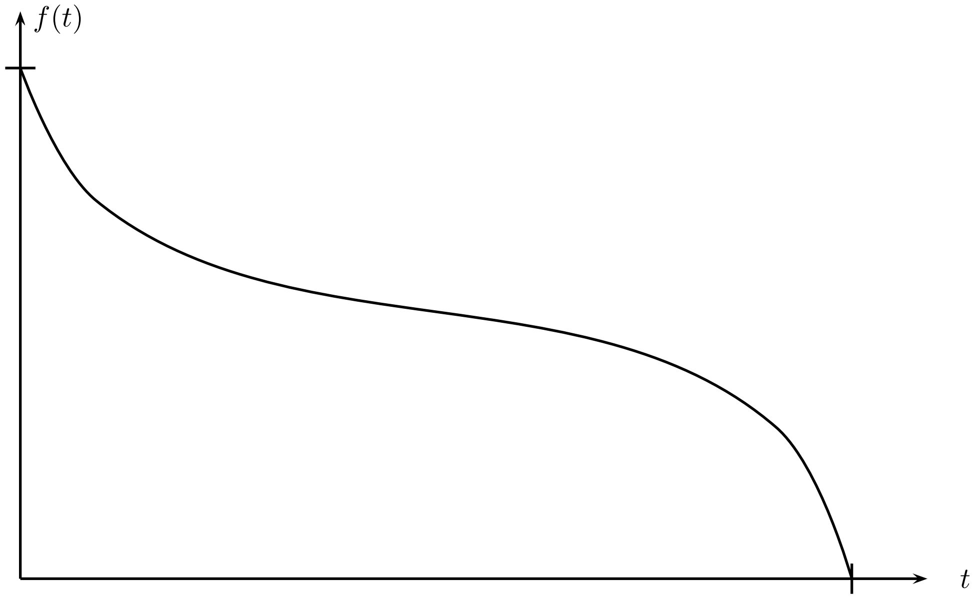
ii. The amount of caffeine that will result in Oren finishing his homework in 21 minutes is approximately 10 milligrams greater than the amount of caffeine that Oren will need in order to finish his homework in 20 minutes.

iii. The rate at which Oren is consuming caffeine 20 minutes into his homework assignment is decreasing by 10 milligrams per minute.

iv. In order to complete tonight's assignment in 19 rather than 20 minutes, Oren needs to consume about 10 milligrams of additional caffeine.

v. If Oren consumes 20 milligrams of caffeine, then he will finish tonight's assignment approximately 10 minutes faster than if he consumes no caffeine.

5. The graph below shows an approximation to the stock price, $P = f(t)$ in dollars, of Lehman Brothers Inc. (LEH) with t measured in months since the stock's highest point in February 2007 to the company's ultimate bankruptcy in September 2008 (t=19).



(a) (2 points) Explain why f is invertible on the indicated domain.

f is invertible on its domain because for each output there is a unique input. Graphically, this is best seen by the fact that the graph passes the horizontal line test.

(b) (3 points) Interpret, in the context of this problem, $f^{-1}(5)$.

$f^{-1}(5)$ is the number of months (since Feb 2007) it took for LEH's stock price to reach \$5.

(c) (4 points) If $\frac{dP}{dt}|_{t=16} = -5$ and $f(16) = 25$, find an equation of the line tangent to the curve at $t = 16$.

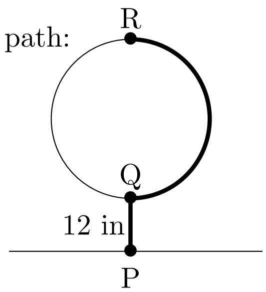
Since we are given the slope and a point, we can use the point-slope equation. Thus, our tangent line equation is $y - 25 = -5(t - 16)$, or equivalently $y = -5t + 105$.

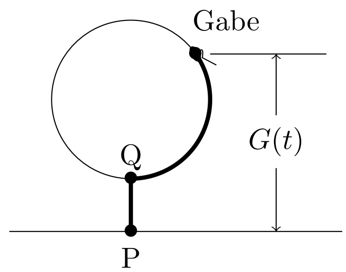
(d) (3 points) Using part (c), what month would your tangent line have predicted LEH's stock price would reach zero?

We need to know what t value yields $y = 0$. From part (c), we get the equation $-25 = -5t + 80$, which has the solution $t = 21$. Now, since t was measured in months since Feb 2007, $t = 21$ corresponds to November 2008 (Lehman actually filed for bankruptcy in September of 2008).

2. [13 points] After Blizzard left Arizona, Gabe the mouse found a large globe (a sphere) to climb. The globe has a diameter of 40 inches and it is attached to a 12-inch-long pole. Gabe starts at the base of the pole at point P. He climbs up to the bottom of the globe at point Q. He then climbs the globe along a semicircle until he stops at the top of the globe at point R (see the diagram below). *Note that the diagram is not drawn to scale.*

Complete path:





a. [8 points] Assume that Gabe walks through the path at a velocity of 3 inches per second. Let $G(t)$ be Gabe's height above the ground (in inches) t seconds after he started his climb at point P. Find a piecewise-defined formula for $G(t)$. *Be sure to include the domain for each piece.*

Solution: From point P to Q: It takes the ant 4 seconds to climb 12 inches at a velocity of 3 inches per second. During that time, the ant climbs at a constant rate of 3 inches per seconds starting at the floor, hence $G(t) = 3t$ for $0 \le t \le 4$.

From point Q to R: The distance L along the semicircle traveled by the ant is $L = \frac{1}{2}(2\pi R)$, where R is the radius of the circle. In this case $R = 20$ inches, then $L = 20\pi$. Hence it takes the ant $T = \frac{L}{3} = \frac{20\pi}{3}$ seconds to go from point Q to R a t a velocity of 3 inches per second. Its height is given by a sinusoidal function with midline at $k = 12 + 20 = 32$, amplitude $A = \frac{1}{2}(40) = 20$, period $P = 2T = \frac{40\pi}{3}$ and a minimum at $(4, 12)$. Hence $G(t) = 32 - 20\cos(B(t-4))$. The constant $B = \frac{2\pi}{P} = \frac{2\pi}{\frac{40\pi}{3}} = \frac{3}{20}$ for $4 \le t \le 4 + T$. Hence

$$G(t) = \begin{cases} 3t & \text{for} \quad 0 \le t \le 4. \\ \\ 32 - 20\cos\left(\frac{3}{20}(t-4)\right) & \text{for} \quad 4 \le t \le 4 + \frac{20\pi}{3} \end{cases}$$

b. [5 points] After climbing the globe, Gabe jumps onto a small ferris wheel. Let $H(t)$ be his height, in inches, above the ground t seconds after Gabe jumped, where

$$H(t) = 12 + 9\cos\left(\frac{\pi}{75}(t - 120)\right).$$

Find the the *smallest* positive value of t at which Gabe's height above the ground is 10.5 inches. Clearly show each step of your algebraic work. Give your answer in *exact* form.

Solution:

$$12 + 9\cos\left(\frac{\pi}{75}(t-120)\right) = 10.5$$

$$\cos\left(\frac{\pi}{75}(t-120)\right) = -\frac{1}{6}$$

$$\frac{\pi}{75}(t-120) = \cos^{-1}\left(-\frac{1}{6}\right) \qquad t_0 = 120 + \frac{75}{\pi}\cos^{-1}\left(-\frac{1}{6}\right)$$

$$\text{(smallest positive)} \qquad t_{ans} = t_0 - P = \frac{75}{\pi}\cos^{-1}\left(-\frac{1}{6}\right) - 30.$$

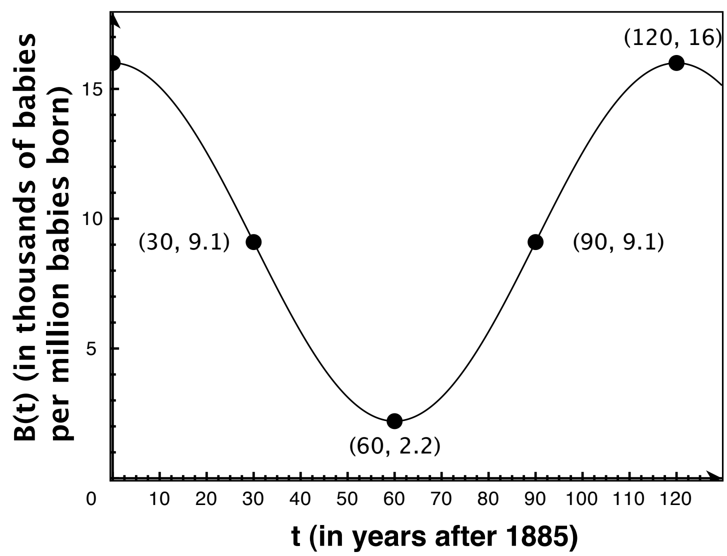
where the period of $H(t)$ is $P = \frac{2\pi}{\frac{\pi}{75}} = 150$.

3. [12 points] The popularity of baby names varies over time; the names that are popular one year may not be popular at all within a few years. The popularity of baby names beginning with the letter I appears to be periodic. In 1885, approximately 16,000 per million babies born had first names beginning with the letter I. Their popularity began decreasing at that time and decreased until 1945, when the number had dropped to a low of 2,200 per million. In 2005 it was back to 16,000 per million babies born.

Let $B(t)$ denote the popularity of names beginning with the letter I, in thousands of babies per million babies born, t years after 1885. Assume that $B(t)$ is a sinusoidal function.

a. [6 points] Sketch the graph of $B(t)$. (Remember to clearly label your graph.)

Solution:



b. [6 points] Find a formula for $B(t)$.

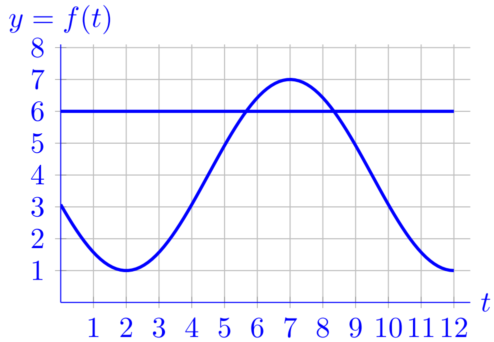
Solution: The amplitude of the function is 6.9, period is 120 years (so $B = \frac{2\pi}{120} = \frac{\pi}{60}$) and the midline is at $y = B(t) = 9.1$. Since $t = 0$ corresponds to a maximum, a possible formula for $B(t)$ is

$$B(t) = 6.9 \cos\left(\frac{\pi}{60} t\right) + 9.1$$

7. [10 points] An apple farmer wants to assess the damage done by a plague to the trees in his orchard. In order to do so, he installs cameras on a couple of small flying robots to film the damage done by the plague to the trees. Let $f(t)$ and $s(t)$ and be the height above the ground (in feet) of the first and second robot t seconds after they started recording.

a. [5 points] Let $f(t) = 4 - 3\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right)$. Find the time(s) at which the first robot is 6 feet above the ground for $0 \le t \le 12$. Your answer(s) should be *exact*. Show all your work.

Solution: From the graph:

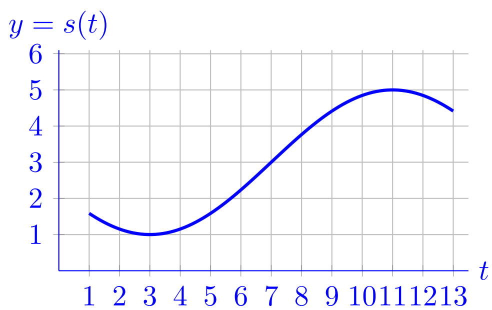


we see that there are two solutions.

$$4 - 3\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right) = 6$$
$$\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right) = -\frac{2}{3}$$
$$\frac{\pi}{5}t - \frac{2\pi}{5} = \cos^{-1}\left(-\frac{2}{3}\right)$$
$$t = \frac{5}{\pi}\left(\cos^{-1}\left(-\frac{2}{3}\right) + \frac{2\pi}{5}\right)$$
$$\frac{\pi}{5}t - \frac{2\pi}{5} = 2\pi - \cos^{-1}\left(-\frac{2}{3}\right)$$
$$t = 2 + \frac{5}{\pi}\left(2\pi - \cos^{-1}\left(-\frac{2}{3}\right)\right)$$
$$t = \frac{5}{\pi}\left(\cos^{-1}\left(-\frac{2}{3}\right) + \frac{2\pi}{5}\right), 2 + \frac{5}{\pi}\left(2\pi - \cos^{-1}\left(-\frac{2}{3}\right)\right)$$

b. [5 points] The graph of the sinusoidal function $s(t)$ is shown below only for $1 \le t \le 13$. Find a formula for $s(t)$.

Solution:



The sinusoidal $s(t) = -A\cos(B(t-h)) + k$ has:

- Amplitude $= 2$ then $A = 2$.
- Midline $y = 3$ then $k = 3$.
- Period $= 16$ then $B = \frac{2\pi}{16} = \frac{\pi}{8}$.
- Horizontal shift $= 3$ then $h = 3$.

Hence $s(t) = -2\cos\left(\frac{\pi}{8}(t-3)\right) + 3$ (other formulas are also possible).

5. [8 points] *Remember to show your work carefully throughout this problem.*
Algie and Cal go on a picnic, arriving at 12:00 noon.

a. [5 points] Five minutes after they arrive, they notice that 5 ants have joined their picnic. More ants soon appear, and after careful study, they determine that the number of ants appears to be increasing by 20% every minute. Find a formula for a function $A(t)$ modeling the number of ants present at the picnic t minutes past noon for $t \geq 5$.

Solution: Since this is an exponential function, there are constants c and b such that $A(t) = cb^t$. We can see immediately that $b = 1.2$. We can then use the fact that we know that $A(5) = 5$ to find c: $c(1.2)^5 = 5$, so $c = 5/(1.2)^5$, which is approximately 2.01. Alternatively, we can use a horizontal shift to say that this is $5(1.2)^{t-5}$.

Answer: $A(t) =$ $5(1.2)^{(t-5)} = \frac{5}{1.2^5}(1.2)^t$

b. [3 points] Algie and Cal notice that their food is, unfortunately, also attracting flies. The number of flies at their picnic t minutes after noon can be modeled by the function $g(t) = 1.8(1.25)^t$. Algie and Cal decide they will end their picnic when there are at least 1000 flies. How long will their picnic last? *Include units.*

Solution: We wish to find t such that $1.8(1.25)^t = 1000$. Then

$$\begin{aligned} 1.8(1.25)^t &= 1000 \\ \ln(1.8(1.25)^t) &= \ln(1000) \\ \ln(1.8) + t\ln(1.25) &= \ln(1000) \\ t\ln(1.25) &= \ln(1000) - \ln(1.8) \\ t &= \ln(1000/1.8)/\ln(1.25) \approx 28.3. \end{aligned}$$

So they end their picnic about 28.3 minutes after noon (when it started).

Answer: **About 28.3 minutes**

6. [6 points] Consider the function

$$R(w) = 2 + (\ln(w))^{\cos(w)}.$$

Use the limit definition of the derivative to write an explicit expression for $R'(\pi)$.
Your answer should not involve the letter R. Do not attempt to evaluate or simplify the limit.
Please write your final answer in the answer box provided below.

Answer: $R'(\pi) =$ $\boxed{\lim_{h\to 0} \frac{\left(2 + (\ln(\pi + h))^{\cos(\pi+h)}\right) - \left(2 + (\ln(\pi))^{\cos(\pi)}\right)}{h}}$

1. [10 points] The following table provides some information on the populations of Detroit and Ann Arbor over time.

Year	1970	2000
Ann Arbor Population (in thousands)	100	114
Detroit Population (in thousands)	1514	

Remember to show your work clearly.

a. [3 points] Suppose that between 1950 and 2000 the population of Ann Arbor grew at a constant rate (in thousands of people per year). Find a formula for a function $A(t)$ modeling the population of Ann Arbor (in thousands of people) t years after 1950.

Solution: Since the population grew at a constant rate, $A(t)$ is a linear function. The slope of the graph of $A(t)$ is

$$\frac{A(50)-A(20)}{50-20} = \frac{114-100}{50-20} = \frac{14}{30} = \frac{7}{15} \approx 0.467\,.$$

Since $A(20) = 100$, we find that $A(t) = 100 + \frac{7}{15}(t-20)$.

Answer: $A(t) =$ $100 + \frac{7}{15}(t-20) = \frac{272}{3} + \frac{7}{15}t \approx 90.67 + 0.467t$

b. [5 points] Suppose that between 1950 and 2000, the population of Detroit decreased by 6% every 4 years. Find a formula for an exponential function $D(t)$ modeling the population of Detroit (in thousands of people) t years after 1950.

Solution: Since $D(t)$ is an exponential function, there are constants a and c so that $D(t) = ca^t$. Since the population of Detroit decreased by 6% every four years, $a^4 = 0.94$. Thus $a = (0.94)^{1/4}$ and $D(t) = c(0.94)^{t/4}$
To solve for c, we note that

$$1514 = D(20) = c(0.94)^{20/4} = c(0.94)^5$$

and thus

$$c = \frac{1514}{(0.94)^5} \approx 2062.94\,.$$

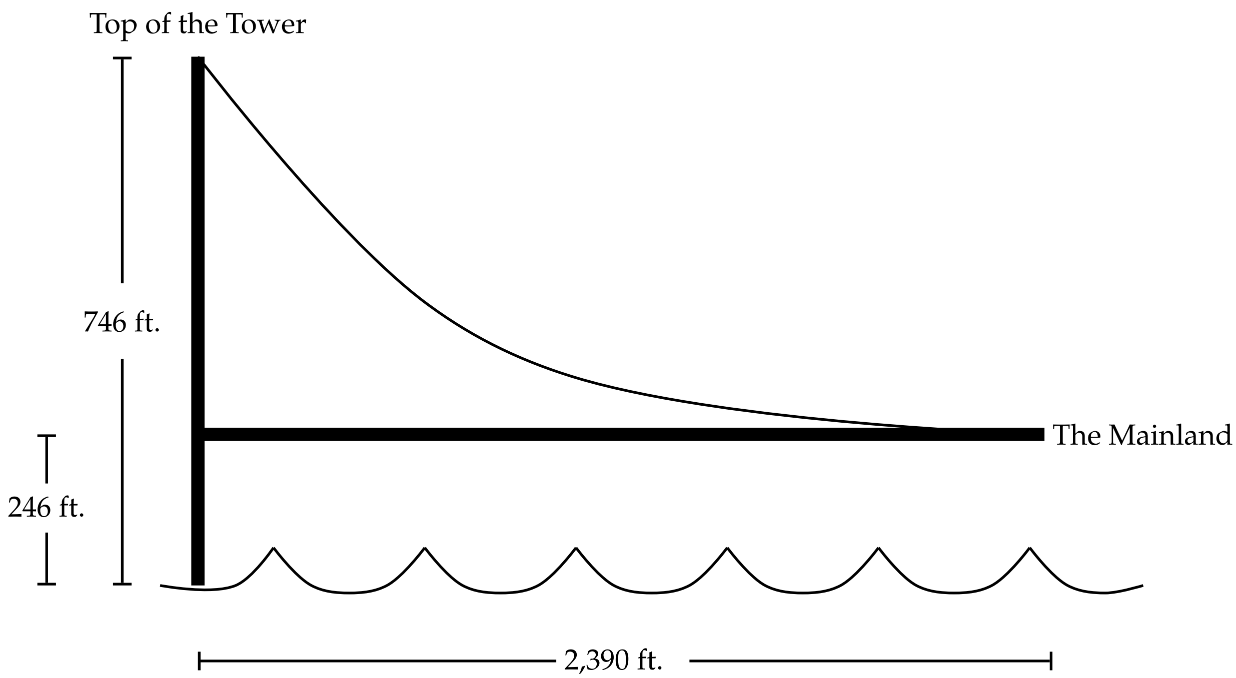
Answer: $D(t) =$ $\frac{1514}{(0.94)^5}(0.94)^{t/4}$ **or** $1514(0.94)^{(t-20)/4}$

c. [2 points] According to your model $D(t)$, what was the population of Detroit in the year 2000? *Include units.*

Solution: $D(50) = \frac{1514}{(0.94)^5}(0.94)^{50/4} \approx 951.89.$

Answer: **about 952 thousand people**

9. San Francisco's famous Golden Gate bridge has two towers which stand 746 ft. above the water, while the bridge itself is only 246 ft. above the water. The last leg of the bridge, which connects to Marin County, is 2,390 ft. long. The suspension cables connecting the top of the tower to the mainland can be modeled by an exponential function. Let $H(x)$ be the function describing the height above the water of the suspension cable as a function of x, the horizontal distance from the tower.



(a) (4 points) Find a formula for $H(x)$.

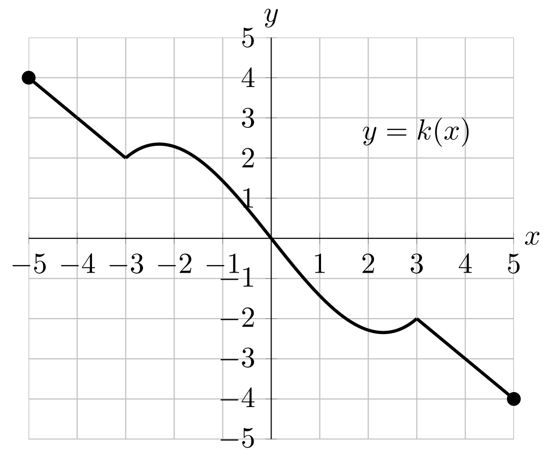
We are looking for a formula of the form $H(x) = H_0 a^x$. We can use the given information to extract the two points which we'll use to find our exponential function: (0,746) and (2390,246). The first of these points gives use the initial value, and from the second we can form the equation $246 = 746a^{2390}$, which can be solved for a. Thus, our final equation is $H(x) = 746(0.9995)^x$, or $H(x) = 746e^{-0.000464x}$.

(b) (4 points) The engineers determined that some repairs are necessary to the suspension cables. They climb up the tower to 400 ft above the bridge, and they need to lay a horizontal walking board between the tower and the suspension cable. How long does the walking board need to be to reach the cable?

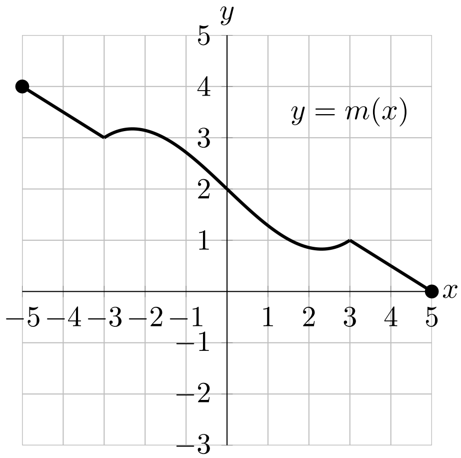
We are looking for an x-value, given that the height up the tower is $246 + 400 = 646$. Thus, we must solve the equation $646 = 746(0.9995)^x$. Solving this equation yields about 287.78 ft, or, if using the second form, $x \approx 310$ ft. (Note: the variance in answers is due to round-off in the representations. Either form is accepted.)

10. [6 points] A part of the graph of a function $k(x)$ with domain $-5 \le x \le 5$ is given below.

In each of the following parts, the corresponding portion of the graph of a function obtained from k by one or more transformations is shown, together with a list of possible formulas for that function. In each case, circle *all* possible formulas for the function shown. *Note that the graphs are not all drawn at the same scale.*



a. [3 points]



A. $\frac{1}{2}k(x) - 2$

B. $\boxed{\frac{1}{2}k(x) + 2}$

C. $-\frac{1}{2}k(-x) - 2$

D. $\boxed{-\frac{1}{2}k(-x) + 2}$

E. $-\frac{1}{2}k(x) - 2$

F. $2k(x) - 2$

G. $2k(x) + 2$

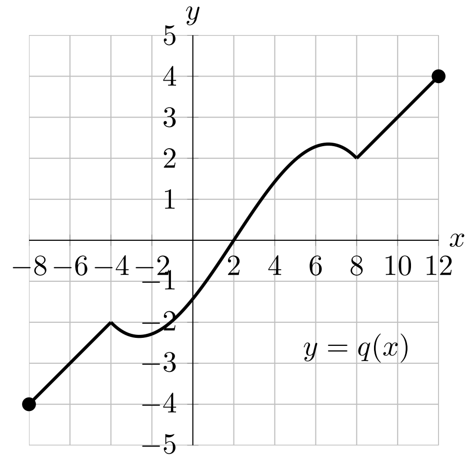
H. $-2k(-x) - 2$

I. $-2k(-x) - 2$

J. $-2k(x) + 2$

K. NONE OF THESE

b. [3 points]



A. $k(2x + 2)$

B. $k(-2x - 2)$

C. $-k(2x + 2)$

D. $k(-2x + 2)$

E. $-k(0.5(x + 2))$

F. $-k(0.5x - 2)$

G. $k(0.5x + 2)$

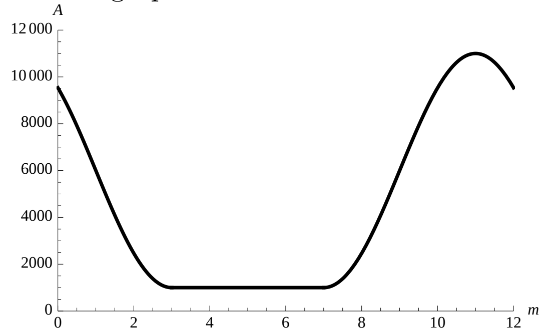
H. $k(0.5(x - 2))$

I. $k(2(x + 1))$

J. $\boxed{-k(0.5(x - 2))}$

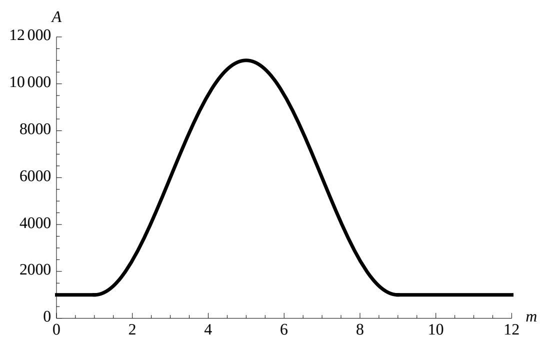
K. NONE OF THESE

8. [18 points] The figure below gives the graph of a function $A = b(m)$. The function is periodic and a full period is shown on the graph.



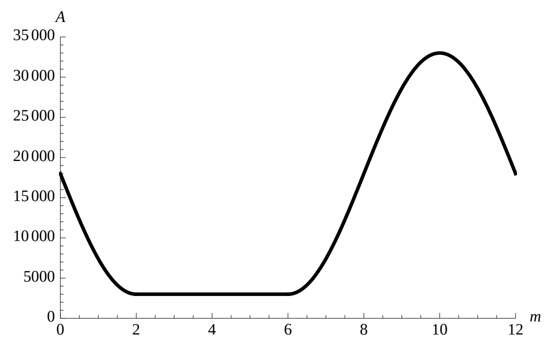
a. [8 points] For each of the following graphs, give an expression for the function depicted in terms of the function b.

$f(m)$



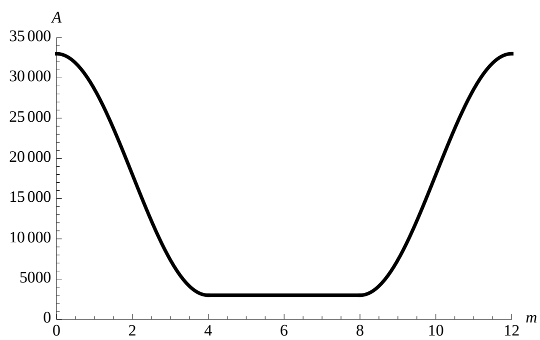
$f(m) = \underline{b(m+6)}$

$g(m)$



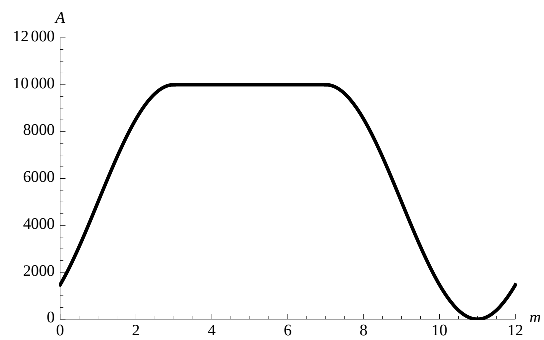
$g(m) = \underline{3b(m+1)}$

$h(m)$



$h(m) = \underline{3b(m-1)}$

$j(m)$



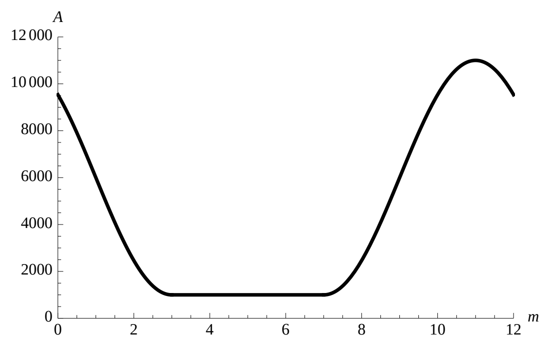
$j(m) = \underline{-b(m) + 11000}$

b. [4 points] The function b from the previous page represents the number of bushels of Michigan-grown organic apples, A, available in Michigan grocery stores as a function of the number of months, m, after January 1. The function $A = b(m)$ is repeated below.

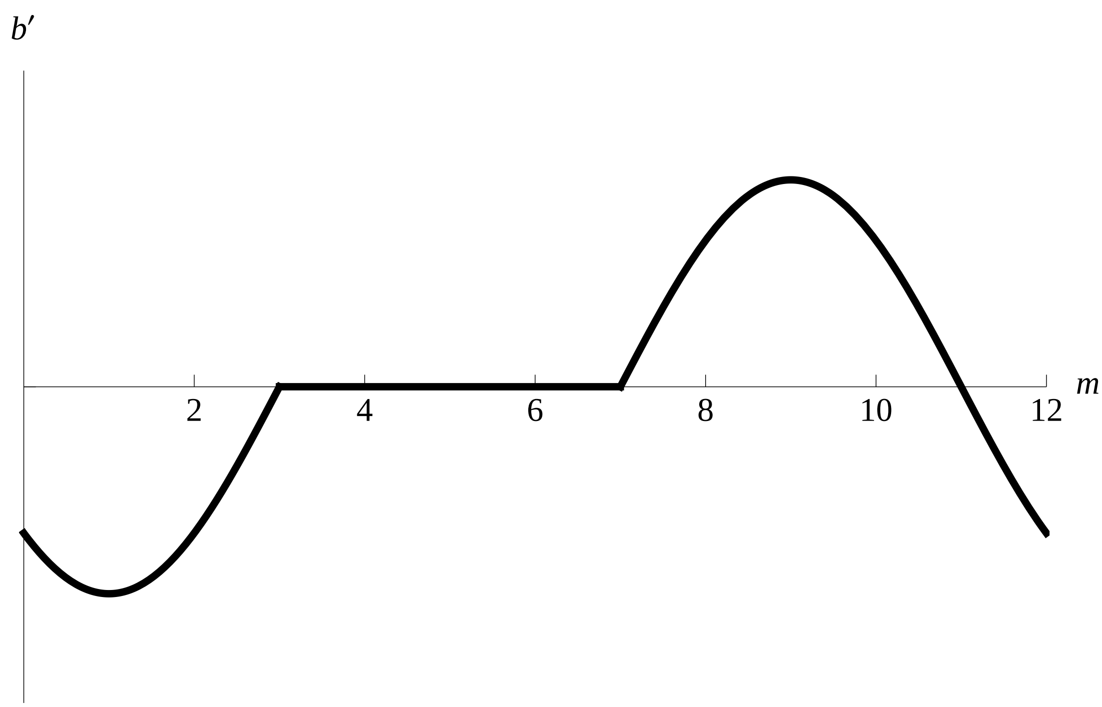
Which of the graphs on the *preceding page* could best correspond to the statement:

"In Washington, the apple growing season starts a month earlier, and the peak grocery store supply is three times as much as in Michigan." Explain your answer.

Solution: The graph of $g(m)$ best corresponds to the statement above. The graph of $g(m)$ has a peak which is three times higher than that of $b(m)$ and the graph has been shifted one unit to the left to signify the growing season beginning one month earlier.



c. [6 points] Using the graph of $b(m)$, repeated above, sketch a well-labeled graph of $b'(m)$.



Note, graphs may differ–answer is not unique.

10. [4 points] Find <u>all</u> real numbers B and positive integers k such that the rational function

$$H(x) = \frac{9 + x^k}{16 - Bx^3}$$

satisfies the following two conditions:

- $H(x)$ has a vertical asymptote at $x = 2$
- $\lim_{x\to\infty} H(x)$ exists.

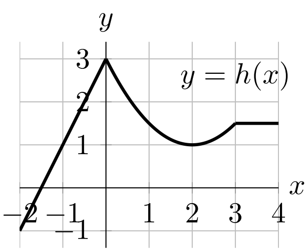
If no such values exist, write NONE.

Justification: In order for $\lim_{x\to\infty} H(x)$ to exist, the degree of the polynomial in the numerator has to be smaller or equal to 3 (the degree of the polynomial in the denominator). In order for $x = 2$ to be a vertical asymptote, you need the denominator to be zero. Hence $16 - B(2^3) = 16 - 8B = 0$ which requires $B = 2$. In this case $H(x) = \frac{9 + x^k}{16 - 2x^3}$ with $k = 1, 2$ or 3. Since $9 + 2^k \neq 0$, then $H(x)$ has a vertical asymptote at $x = 2$ when $B = 2$.

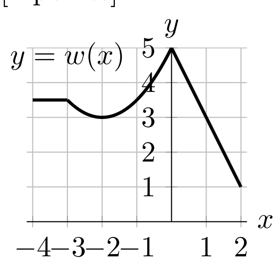
Answer: $B =$ **2** **Answer:** $k =$ **1, 2, or 3**

11. [4 points] A part of the graph of a function $h(x)$ is given below.

In each of the following parts, the corresponding portion of the graph of a function obtained from h by one or more transformations is shown, together with a list of possible formulas for that function. In each case choose the one correct formula for the function shown. *Note that the graphs are not all drawn at the same scale.*



a. [2 points]



A. $h(-x) - 2$

B. $-h(x) - 2$

C. $-h(x) + 2$

D. $\boxed{h(-x) + 2}$

E. $-h(x + 2)$

F. $-h(x - 2)$

G. $-h(-x + 2)$

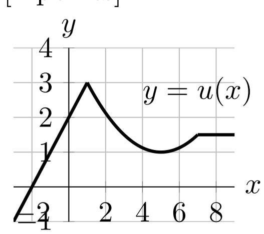
H. $-h(-x - 2)$

I. $h(-x + 2)$

J. $h(-x - 2)$

K. NONE OF THESE

b. [2 points]



A. $h(0.5x + 1)$

B. $h(0.5x - 1)$

C. $\boxed{h(0.5(x - 1))}$

D. $h(0.5(x + 1))$

E. $h(0.5(x - 0.5))$

F. $h(2x - 1)$

G. $h(2x + 1)$

H. $h(2(x - 1))$

I. $h(2(x + 1))$

J. $h(0.5(x + 0.5))$

K. NONE OF THESE

8. [10 points] Let A and B be **positive** constants. The rational functions $y = P(x)$ and $y = Q(x)$ are given by the following formulas:

$$P(x) = \frac{5x(x-2)(Ax+1)^2}{(3x^2+B)(x^2-9)} \qquad Q(x) = \frac{P(x)(x-3)}{x-2}$$

Your answers below may depend on the constants A and B and should be in exact form. You do not need to show your work.

a. [3 points] Find the zeros of the function $y = P(x)$. If P has no zeros write "NONE".

Solution: Setting $5x(x-2)(Ax+1)^2 = 0$ you get $x = 0$, $x - 2 = 0$ and $(Ax+1)^2 = 0$. This yields $x = 0$, 2 and $-\frac{1}{A}$.

Answer: $x = 0$, 2 and $-\frac{1}{A}$.

b. [2 points] What is the domain of $P(x)$?

Solution: The only points not in the domain of $P(x)$ are the solutions to $(3x^2+B)(x^2-9) = 0$. Solving $x^2 - 9 = 0$ we get $x = \pm 3$. If we set $3x^2 + B = 0$, we get $x^2 = -\frac{B}{3} < 0$. This is not possible for any value of x. Then the only solutions are $x = \pm 3$.

Answer: $x \neq -3, 3$.

c. [2 points] Find the *equation(s)* of the horizontal asymptote(s) of $y = P(x)$. If it has no horizontal asymptotes, write "NONE".

Solution: To find the end behavior of $P(x)$ we need to find the leading coefficient of the numerator and the denominator. The leading term of the numerator is $5x(x-2)(Ax+1)^2$ is $5x(x)(Ax)^2 = 5A^2x^4$. The leading term of $(3x^2+B)(x^2-9)$ is $(3x^2)(x^2) = 3x^4$. Hence $\lim_{x\to\infty} P(x) = \lim_{x\to\infty} \frac{5A^2x^4}{3x^4} = \frac{5A^2}{3}$. This limit is the same as $\lim_{x\to-\infty} P(x)$.

Answer: $y = \frac{5A^2}{3}$

d. [3 points] If $A = 1$, find the values of c where $\lim_{x\to c} Q(x)$ does not exist. If no such values of c exist, write "NONE".

Solution: $\lim_{x\to c} Q(x)$ exists for all c in the domain of $Q(x) = \frac{5x(x-2)(Ax+1)^2(x-3)}{(3x^2+B)(x^2-9)(x-2)}$. Hence we need to check the limits at $c = -3$, 2 and 3. At $c = 2$, $\lim_{x\to 2} Q(x) = \frac{-2(2A+1)^2}{B+12}$ and at $c = 3$, $\lim_{x\to 3} Q(x) = \frac{15(3A+1)^2}{6(B+27)}$ (both of these points are holes in the graph of $Q(x)$). At $c = -3$, $Q(x)$ has a vertical asymptote hence $\lim_{x\to -3} Q(x)$ does not exist.

Answer: $c = -3$

1. [11 points] Brianna rides her unicycle north from her home to the grocery store and back again. The differentiable function $r(t)$ represents Brianna's distance in meters from her home t minutes after she leaves the house. Some values of $r(t)$ are shown in the table below.

t	0	1	5	7	10	12	14	16	17
$r(t)$	0	180	1050	1420	1425	980	570	220	0

a. [2 points] What was Brianna's average velocity between times $t = 7$ and $t = 12$? Include units.

Solution: Average velocity$=\dfrac{980-1420}{12-7} = \dfrac{}{5} = -88$ **Answer:** -88 meters per minute.

b. [2 points] Approximate the value of $r'(14)$. Include units.

Solution: $r'(14) \approx \dfrac{220-570}{2} = -175$ **Answer:** -175 meters per minute.

c. [3 points] For which of the following time interval(s) is it possible for $r(t)$ to be concave up on the entire interval? Circle all correct choices.

Solution: Computing average rate of changes in consecutive subintervals we see that

Intervals	[1,5]	[5,7]	[10,12]	[12,14]
Average rate of change	$\frac{870}{4} = 217.5$	$\frac{370}{2} = 185$	$-\frac{445}{2} = -222.5$	$-\frac{410}{2} = -205$

Since the average rate of change only increases on $[10, 14]$, then it is possible that $r(t)$ is concave up on $[\mathbf{10}, \mathbf{14}]$.

Use the following additional information about Brianna's ride to answer the questions below:

- The grocery store is 1430 meters away from Brianna's home.
- It takes Brianna 8 minutes to get to the store.
- On her way to the store, Brianna does not stop at all. On her way back, she only stops once at a traffic light, which is 250 meters from her home.

d. [2 points] For which of the following time interval(s) is $r'(t)$ equal to 0 for some value of t in that interval? Circle all correct choices.

Solution: Based on the information given $r'(t) \neq 0$ on $[1, 5]$ and $[10, 12]$. $r'(8) = 0$ since it takes 8 minutes to get to the store. Since she stops on her way back, then $r'(t) = 0$ for $14 \leq t \leq 16$.

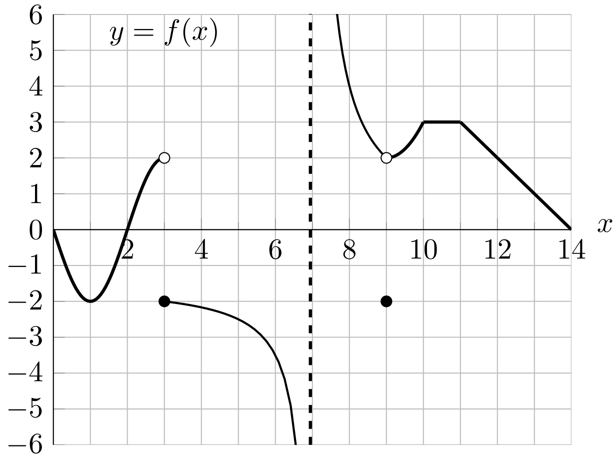
[1,5] $\boxed{[5,10]}$ [10,12] $\boxed{[12,16]}$ NONE OF THESE

e. [2 points] For which of the following time interval(s) is $r'(t)$ negative for some value of t in that interval? Circle all correct choices.

Solution: The derivative of $r(t)$ is negative on her way back.

[1,5] $\boxed{[5,10]}$ $\boxed{[10,12]}$ $\boxed{[12,16]}$ NONE OF THESE

5. [15 points]
Below is a portion of the graph of an **odd** function $f(x)$, and the formula for a function $g(x)$. Note that $f(x)$ is linear for $11 < x < 14$.



$$g(x) = \frac{x^4 + 1}{e^{x^2}}$$

In the following parts, evaluate each of the given quantities. If the value does not represent a real number (including the case of limits that diverge to ∞ or $-\infty$), write DNE or "does not exist." You do not need to show work in this problem. Give your answers in **exact form**.

a. [2 points] $g(f(2))$

Answer: $=$ **1**

b. [2 points] $\lim_{x\to 7} f(x)$

Answer: $=$ **DNE**

c. [2 points] $\lim_{x\to -1} \Big(f(x) + g(x)\Big)$

Answer: $=$ $2 + \frac{2}{e}$

d. [2 points] $\lim_{x\to \infty} g(x)$

Answer: $=$ **0**

e. [2 points] $\lim_{h\to 0} \dfrac{f(12+h) - 2}{h}$

Answer: $=$ **-1**

f. [2 points] $\lim_{x\to -9} f(x)$

Answer: $=$ **-2**

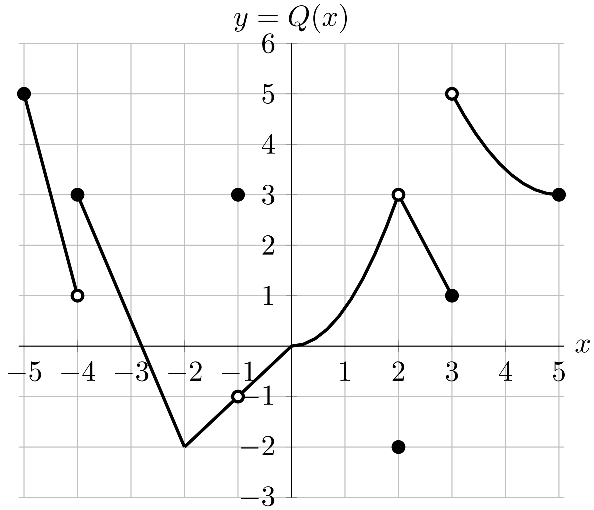
g. [2 points] $\lim_{x\to 11^+} f(f(x))$

Answer: $=$ **2**

h. [1 point] $\lim_{x\to 3} g(f(x))$

Answer: $=$ $\frac{17}{e^4}$

4. [14 points] The graph of a function $Q(x)$ with domain $[-5, 5]$ is shown below.



a. [2 points] On which of the following intervals is $Q(x)$ invertible? Circle all that are true.

$[-4, -1]$ $\qquad$ $[-2, 3]$ $\qquad$ $[2, 5]$ $\qquad$ $[-2, 2]$ $\qquad$ NONE OF THESE.

b. [8 points] Find the numerical value of the following mathematical expressions. If the answer cannot be determined with the information given, write "NI". If any of the quantities does not exist, write "DNE".

i) Find $\lim_{x \to -1} Q(x)$

Solution:

i) **Answer:** -1.

ii) Find $\lim_{w \to 2} Q(Q(w))$

Solution:

ii) **Answer:** 1.

iii) Find $\lim_{h \to 0} \dfrac{Q(-3+h) - Q(-3)}{h}$

Solution:

iii) **Answer:** -2.5.

iv) Find $\lim_{x \to \infty} Q\left(\dfrac{1}{x} + 3\right)$

Solution:

iv) **Answer:** 5.

v) Find $\lim_{x \to \frac{1}{3}} xQ(3x - 5)$

Solution:

v) **Answer:** DNE.

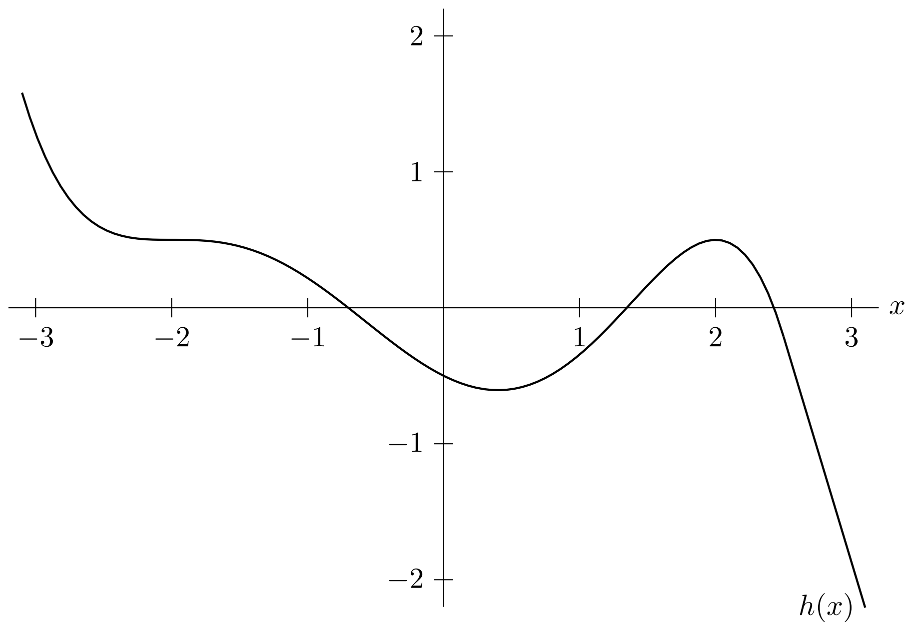
c. [2 points] For which values of $-5 < x < 5$ is the function $Q(x)$ not continuous?

Solution: $x = -4, -1, 2, 3.$

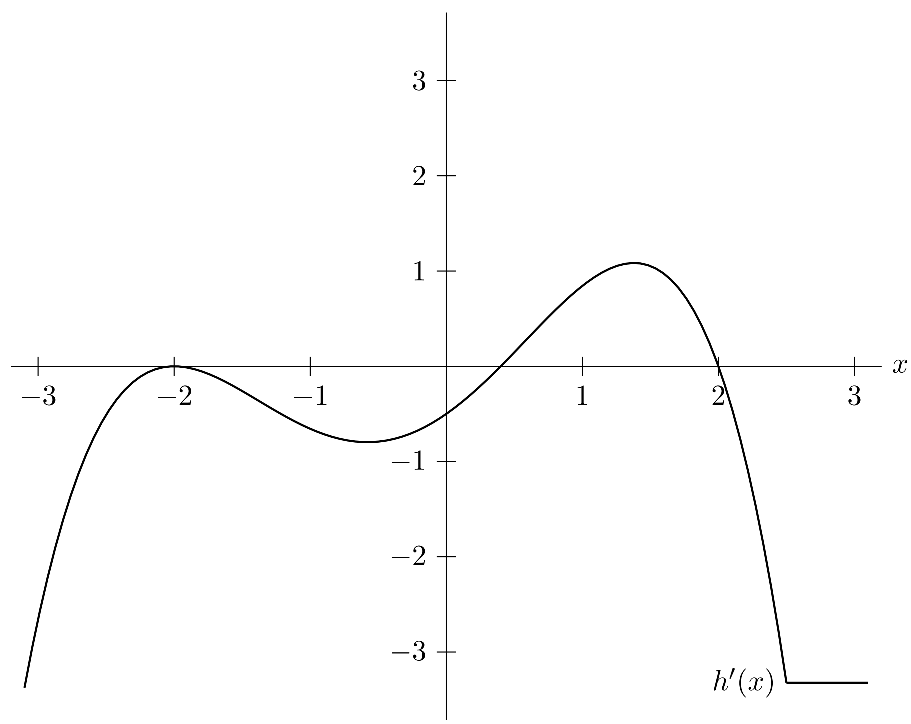
d. [2 points] For which values of $-5 < p < 5$ is $\lim_{x \to p^-} Q(x) \neq Q(p)$?

Solution: $p = -4, -1, 2.$

9. [10 points] Given below is the graph of a differentiable function $h(x)$ which is linear for $x > 2.5$. On the second set of axes, sketch a possible graph of $h'(x)$. Be sure your graph is drawn carefully.



Solution:



9. [8 points] On the axes provided below, sketch the graph of a single function f satisfying all of the following:

- $f''(x) > 0$ for $x < -2$.
- The graph of f has a vertical asymptote at $x = -2$.
- $f'(-1) = -3$
- $\lim_{x \to 0} f(x) = 2$
- $f(0) = -2$
- f is continuous but not differentiable at $x = 1$.
- $f'(x) > 0$ for $x > 3$.
- $\lim_{x \to \infty} f(x) = 4$

Remember to clearly label your graph.

Solution: A possible graph of the function is shown below: