Math 115 — Practice for Exam 1

Generated October 19, 2025

UMID: **SOLUTIONS** Initials: ______

Instructor: ______________________ Section Number: __________

1. This exam has 15 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You may not use a calculator. You are allowed one double-sided 8×11 inch page of handwritten notes.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. You must use the methods learned in this course to solve all problems.
7. You are responsible for reading and following all directions provided for each exam in this course.

Semester	Exam	Problem	Name	Points	Score
Winter 2015	2	9	prison break 2	9	
Fall 2018	1	1	unicycle	11	
Winter 2023	2	10		12	
Winter 2015	2	6		14	
Fall 2023	2	3		4	
Winter 2020	2	4	honey	12	
Winter 2020	1	7	sheep and cows	11	
Fall 2016	2	4		8	
Winter 2023	2	1		10	
Winter 2024	1	3		5	
Fall 2016	1	3		10	
Winter 2018	1	7	orchard robots	10	
Winter 2016	1	11		11	
Fall 2012	1	5		12	
Winter 2017	1	4	pests	10	
Total				149	

Recommended time (based on points): 142 minutes

9. [9 points] Elphaba and Walt are planning to break out of prison. They would like to escape no later than 20 hours after devising their plan, and they would like to attempt their escape during the noisiest part of the day. Let $N(t)$ be the noise level (in decibels) in the prison t hours after Elphaba and Walt have devised their escape plan.
On the interval $[0, 20]$, a formula for $N(t)$ is given by

$$N(t) = 60 + 1.01^{p(t)} \quad \text{where} \quad p(t) = \frac{1}{3}t^3 - 9t^2 + 56t + 200.$$

a. [8 points] Find the values of t that minimize and maximize $N(t)$ on the interval $[0, 20]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema.

Solution: Since $N(t)$ is continuous on the interval $[0, 20]$, we can apply the Extreme Value Theorem and compare the values of $N(t)$ at the critical points and endpoints of the interval.
We first need to find the critical points in this interval. Taking the derivative of $N(t)$ and setting it equal to zero we have $N'(t) = \ln(1.01)p'(t)(1.01)^{p(t)} = 0$ so critical points occur when $0 = p'(t) = t^2 - 18t + 56$. Solving we determine that the only critical points of $N(t)$ occur at $t = 4$ and $t = 14$, which are both in the interval $[0, 20]$.
To find the global extrema we need to evaluate the function at $t = 0, 4, 14, 20$. We find $N(0) \approx 67.316$, $N(4) \approx 80.0528$, $N(14) \approx 63.819$ and $N(20) \approx 106.874$. Choosing the largest and smallest values, by the Extreme Value Theorem, we see that the global minimum occurs at $t = 14$ and the global maximum occurs at $t = 20$.

(For each answer blank below, write NONE in the answer blank if appropriate.)

Answer: global min(s) at $t =$ 14

Answer: global max(es) at $t =$ 20

b. [1 point] As mentioned above, Elphaba and Walt would like to escape no later than 20 hours after devising their plan, and they would like to escape during the noisiest part of the day. When should Elphaba and Walt attempt their escape?

Answer: They should try to escape 20 hours after devising their plan.

1. [11 points] Brianna rides her unicycle north from her home to the grocery store and back again. The differentiable function $r(t)$ represents Brianna's distance in meters from her home t minutes after she leaves the house. Some values of $r(t)$ are shown in the table below.

t	0	1	5	7	10	12	14	16	17
$r(t)$	0	180	1050	1420	1425	980	570	220	0

a. [2 points] What was Brianna's average velocity between times $t = 7$ and $t = 12$? Include units.

Solution: Average velocity$=\dfrac{980-1420}{12-7} = \dfrac{}{5} = -88$ **Answer:** -88 meters per minute.

b. [2 points] Approximate the value of $r'(14)$. Include units.

Solution: $r'(14) \approx \dfrac{220-570}{2} = -175$ **Answer:** -175 meters per minute.

c. [3 points] For which of the following time interval(s) is it possible for $r(t)$ to be concave up on the entire interval? Circle all correct choices.

Solution: Computing average rate of changes in consecutive subintervals we see that

Intervals	[1,5]	[5,7]	[10,12]	[12,14]
Average rate of change	$\frac{870}{4} = 217.5$	$\frac{370}{2} = 185$	$-\frac{445}{2} = -222.5$	$-\frac{410}{2} = -205$

Since the average rate of change only increases on $[10, 14]$, then it is possible that $r(t)$ is concave up on $[\mathbf{10}, \mathbf{14}]$.

Use the following additional information about Brianna's ride to answer the questions below:

- The grocery store is 1430 meters away from Brianna's home.
- It takes Brianna 8 minutes to get to the store.
- On her way to the store, Brianna does not stop at all. On her way back, she only stops once at a traffic light, which is 250 meters from her home.

d. [2 points] For which of the following time interval(s) is $r'(t)$ equal to 0 for some value of t in that interval? Circle all correct choices.

Solution: Based on the information given $r'(t) \neq 0$ on $[1, 5]$ and $[10, 12]$. $r'(8) = 0$ since it takes 8 minutes to get to the store. Since she stops on her way back, then $r'(t) = 0$ for $14 \leq t \leq 16$.

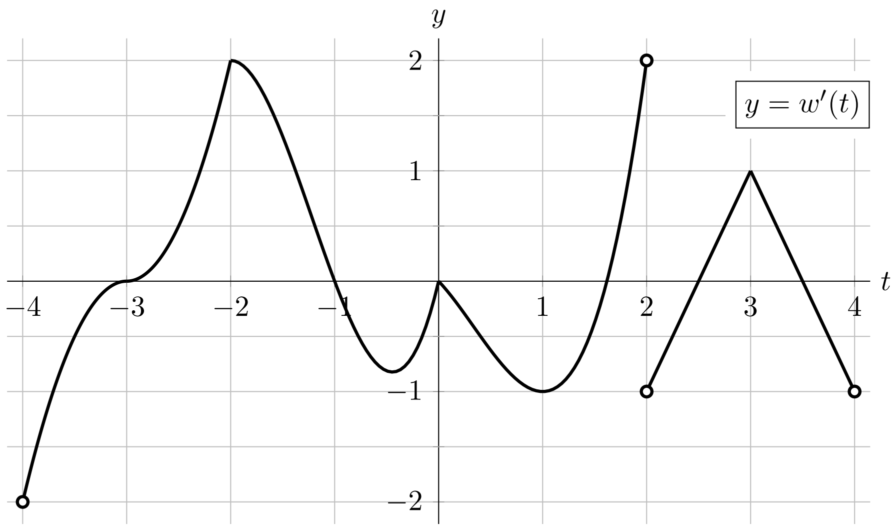
[1,5] $\boxed{[5,10]}$ [10,12] $\boxed{[12,16]}$ NONE OF THESE

e. [2 points] For which of the following time interval(s) is $r'(t)$ negative for some value of t in that interval? Circle all correct choices.

Solution: The derivative of $r(t)$ is negative on her way back.

[1,5] $\boxed{[5,10]}$ $\boxed{[10,12]}$ $\boxed{[12,16]}$ NONE OF THESE

10. [12 points] Suppose $w(t)$ is a continuous function, defined on the interval $(-4, 4)$. A graph of the **derivative** $w'(t)$ is given below.



a. [2 points] Circle all points below that are critical points of $w(t)$.

$\boxed{t = -3}$ $\quad t = -2 \quad t = 1 \quad$ $\boxed{t = 2}$ $\quad t = 3$ NONE OF THESE

b. [2 points] Circle all points below that are critical points of $w'(t)$.

$\boxed{t = -3}$ $\boxed{t = -2}$ $\boxed{t = 1}$ $\quad t = 2 \quad$ $\boxed{t = 3}$ NONE OF THESE

c. [2 points] Circle all points below that are local minima of $w(t)$.

$\boxed{t = -3}$ $\quad t = -2 \quad t = -1 \quad t = 1 \quad t = 2$ NONE OF THESE

d. [2 points] Circle all points below that are local maxima of $w(t)$.

$t = -3 \quad t = -2 \quad$ $\boxed{t = -1}$ $\quad t = 1 \quad$ $\boxed{t = 2}$ NONE OF THESE

e. [2 points] Circle all points below that are inflection points of $w(t)$.

$t = -3 \quad$ $\boxed{t = -2}$ $\boxed{t = 1}$ $\quad t = 2 \quad$ $\boxed{t = 3}$ NONE OF THESE

f. [1 point] Circle all points below that are global maxima of $w'(t)$ on the interval $(-4, 4)$.

$t = -4 \quad$ $\boxed{t = -2}$ $\quad t = 1 \quad t = 2 \quad t = 3$ NONE OF THESE

g. [1 point] Circle all points below that are global minima of $w'(t)$ on the interval $(-4, 4)$.

$t = -4 \quad t = -2 \quad t = 1 \quad t = 2 \quad t = 3$ $\boxed{\text{NONE OF THESE}}$

6. [14 points] Let p be a function such that $p''(x)$ is defined for all real numbers x. A table of some values of $p'(x)$ is given below.

x	-9	-5	-1	3	7	11
$p'(x)$	-3	0	-4	0	2	1

Assume that p' is either always strictly decreasing or always strictly increasing between consecutive values of x shown in the table.

For each of the questions below, circle ALL of the appropriate choices. If none of the choices are correct, circle NONE OF THESE.

a. [2 points] At which, if any, of the following values of x does $p(x)$ definitely have a local maximum in the interval $-9 < x < 11$?

-5 $\quad -1$ $\quad 3$ $\quad 7$ $\quad$ $\boxed{\text{NONE OF THESE}}$

Solution: The only critical points of $p(x)$ in this interval are at $x = -5$ and $x = 3$. We now classify these critical points. $p(x)$ does not have a local extremum at $x = -5$ since $p'(x)$ is negative both immediately before and after $x = -5$. $p(x)$ has a local minimum at $x = 3$ since the sign of $p'(x)$ changes from negative to positive there.

b. [2 points] At which, if any, of the following values of x does $p(x)$ definitely attain its global minimum on the interval $-9 \le x \le 11$?

-9 $\quad -5$ $\quad -1$ $\quad \boxed{3}$ $\quad 7$ $\quad 11$ $\quad$ NONE OF THESE

Solution: $p(x)$ is decreasing for $-9 \le x \le 3$ and increasing for $3 \le x \le 11$.

c. [2 points] At which, if any, of the following values of x does $p'(x)$ (the <u>derivative</u> of $p(x)$) definitely attain its global maximum on the interval $-9 \le x \le 11$?

-9 $\quad -5$ $\quad -1$ $\quad 3$ $\quad \boxed{7}$ $\quad 11$ $\quad$ NONE OF THESE

Solution: Since $p'(x)$ is always increasing of decreasing between points in the table, the global max must occur at one of the values of x in the table. Realizing this, we need to choose the value of x in the table that gives the largest value of $p'(x)$, which is $x = 7$.

d. [3 points] On which of the following intervals is $p(x)$ definitely always concave up?

$\boxed{-9 < x < -5}$ $\quad -5 < x < -1$ $\quad \boxed{-1 < x < 3}$ $\quad \boxed{3 < x < 7}$ $\quad 7 < x < 11$ $\quad$ NONE OF THESE

Solution: The function $p(x)$ is concave up whenever the derivative $p'(x)$ is increasing.

e. [3 points] At which, if any, of the following values of x does $p(x)$ definitely have an inflection point in the interval$-9 < x < 11$?

$\boxed{-5}$ $\quad \boxed{-1}$ $\quad 3$ $\quad \boxed{7}$ $\quad$ NONE OF THESE

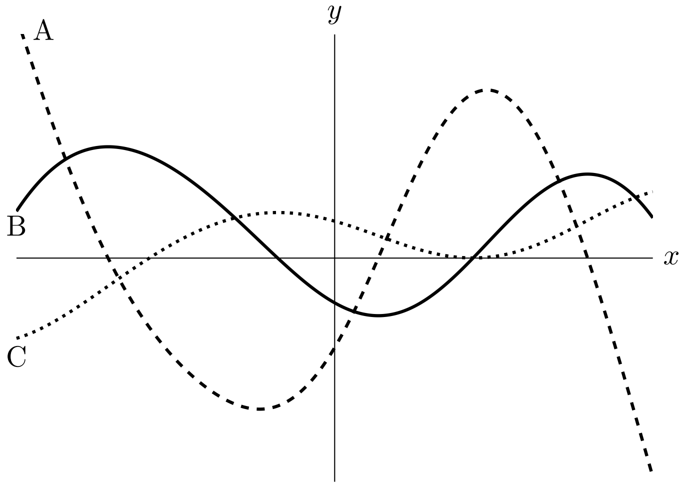
Solution: We are looking for places where $p'(x)$ changes from increasing to decreasing or vice versa.

f. [2 points] Which, if any, of the following must be true?

$\boxed{p''(7) \ge p''(-3)}$ $\quad p''(7) = p''(-3)$ $\quad p''(7) \le p''(-3)$ $\quad$ NONE OF THESE

Solution: The function $p'(x)$ is increasing between 3 and 7 and decreasing between 7 and 11 so $p''(7) = 0$. The function $p'(x)$ is decreasing between -5 and -1, so $p''(-3) \le 0$.

3. [4 points] Shown below are portions of the graphs of the functions $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



Answer: $f(x)$: C

$f'(x)$: B

$f''(x)$: A

4. [8 points] Suppose $f(x)$ and $g(x)$ are functions that have exactly the same four critical points, namely at $x = 1$, $x = 3$, $x = 5$, and $x = 7$. Note that f and g have **no other** critical points beyond these four. Assume the first and second derivatives of $f(x)$ and $g(x)$ exist everywhere.

The table below shows some values of $f'(x)$ and $g''(x)$ at certain inputs. Note that the table gives values of the **first derivative** of $f(x)$ and the **second derivative** of $g(x)$.

x	0	1	2	3	4	5	6	7	8
$f'(x)$	3	0	−1	0	1	0	2	0	?
$g''(x)$	?	0	−1	−4	?	0	?	2	1

a. [4 points] Use the table to classify each critical point of f as a local minimum, maximum, or neither of f. Circle your answer. If there is not enough information to decide, circle NEI.

i. $x = 1$ is a LOCAL MIN of f $\boxed{\text{LOCAL MAX of } f}$ NEITHER NEI

ii. $x = 3$ is a $\boxed{\text{LOCAL MIN of } f}$ LOCAL MAX of f NEITHER NEI

iii. $x = 5$ is a LOCAL MIN of f LOCAL MAX of f $\boxed{\text{NEITHER}}$ NEI

iv. $x = 7$ is a LOCAL MIN of f LOCAL MAX of f NEITHER $\boxed{\text{NEI}}$

b. [4 points] Use the table to classify each critical point of g as a local minimum, maximum, or neither of g. Circle your answer. If there is not enough information to decide, circle NEI.

i. $x = 1$ is a LOCAL MIN of g LOCAL MAX of g NEITHER $\boxed{\text{NEI}}$

ii. $x = 3$ is a LOCAL MIN of g $\boxed{\text{LOCAL MAX of } g}$ NEITHER NEI

iii. $x = 5$ is a LOCAL MIN of g LOCAL MAX of g $\boxed{\text{NEITHER}}$ NEI

iv. $x = 7$ is a $\boxed{\text{LOCAL MIN of } g}$ LOCAL MAX of g NEITHER NEI

Solution: Part **a.** follows from the First Derivative Test, and most of **b.** from the Second Derivative Test. For **b.**(iii.), note that g must be decreasing on both $(3, 5)$ and $(5, 7)$ since $x = 5$ is the only critical point of g on $(3, 7)$ and we have $g''(3) < 0$ but $g''(7) > 0$.

Note: exam problem numbering is off by 1

5. [12 points] Isabelle is a bee keeper who wants to sell honey at the local farmers market. Let $y = H(d)$ be the amount of honey, in pounds, that Isabelle will sell in a month if she charges d dollars per pound of honey. The functions $H(d)$ and $H'(d)$ are defined and differentiable for all $d \geq 0$. Some values are given in the table below.

d	5.00	5.75	6.50	7.25	8.00	8.75
$H(d)$	59	52	46	38	29	23
$H'(d)$	-10.4	-9.1	-7.8	-11.0	-12.2	-7.6

Assume that $H(d)$ is decreasing and that between each pair of consecutive values of d given in the table, $H'(d)$ is either always increasing or always decreasing.

a. [3 points] Write a formula for the linear approximation $L(d)$ of $H(d)$ near $d = 6.50$, and use it to estimate the amount of honey, in pounds, Isabelle will sell if she charges \$6.30 per pound.

Answer: $L(d) =$ $-7.8(d - 6.5) + 46$

Answer: $\approx$ \$47.56

b. [2 points] Is your estimate from the previous part an overestimate, an underestimate, neither, or is there not enough information to decide? Briefly explain your answer.

Solution: At $d = 6.3$, $H'(d)$ is increasing. Thus $H(d)$ is concave up, and so the linear approximation is an underestimate.

c. [3 points] Write a formula for the linear approximation $K(y)$ of $(H^{-1})(y)$ near $y = 31$.

Solution: Not enough information due to a typo; all students earned these 3 points on the exam.

d. [2 points] Use the table to approximate $H''(8.75)$.

Answer: $H''(8.75) \approx$ $\frac{H'(8.75)-H'(8)}{8.75-8} \approx 6.133$

e. [2 points] The hypotheses of the Mean Value Theorem are satisfied for $H(d)$ on the interval $[5.00, 5.75]$. The conclusion of the theorem then tells you that there is a c in the interval $[5, 5.75]$ so that

$H'(c)$ $=$ $\frac{H(5.75)-H(5)}{5.75-5} \approx -9.33$

7. [11 points] Falco decides to raise sheep and cows on his farm.

- Let $W(p)$ be the amount of wool, in pounds per year, produced by a sheep who was fed p pounds of food per day.
- Let $M(p)$ be the amount of milk, in thousands of gallons per year, produced by a cow who was fed p pounds of food per day.

The functions $W(p)$ and $M(p)$ are differentiable and invertible.

a. [2 points] Use a complete sentence to give a practical interpretation of the the equation

$$W^{-1}(28) = 10.$$

Solution: A sheep fed 10 pounds of food per day produced 28 pounds of wool per year.

b. [3 points] Write a single equation representing the following statement in terms of the functions W, M, and/or their inverses:

A sheep that produced 12 pounds of wool per year was fed 5 fewer pounds of food per day than a cow that produced 2430 gallons of milk per year.

Answer: $M^{-1}(2.43) = W^{-1}(12) + 5$

c. [3 points] Complete the following sentence to give a practical interpretation of the equation

$$M'(23) = 0.15.$$

If Falco feeds a cow 22.4 pounds of food per day instead of 23 pounds of food per day, then ...

Solution: ...the cow will produce approximately 90 fewer gallons of milk per year.

d. [3 points] Circle the one sentence that gives a valid interpretation of the equation

$$(W^{-1})'(12) = 0.7.$$

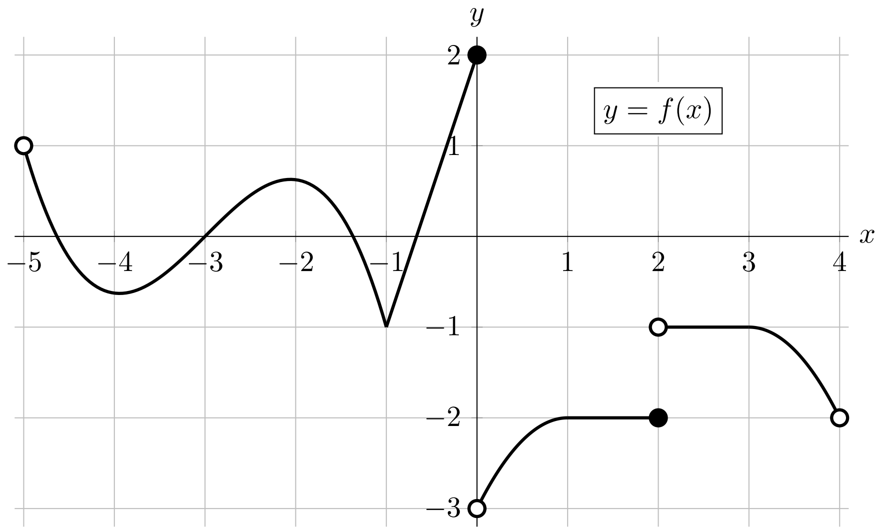
i. To increase a sheep's wool production from 12 pounds per year to 12.6 pounds per year, Falco should feed it about 0.42 extra pounds of food per day.

ii. To increase a sheep's wool production from 12 pounds per year to 12.1 pounds per year, Falco should feed it approximately 0.7 more pounds of food per day.

iii. A sheep that is fed 12.3 pounds of food per day instead of 12 pounds of food per day will produce approximately 0.21 additional pounds of wool per year.

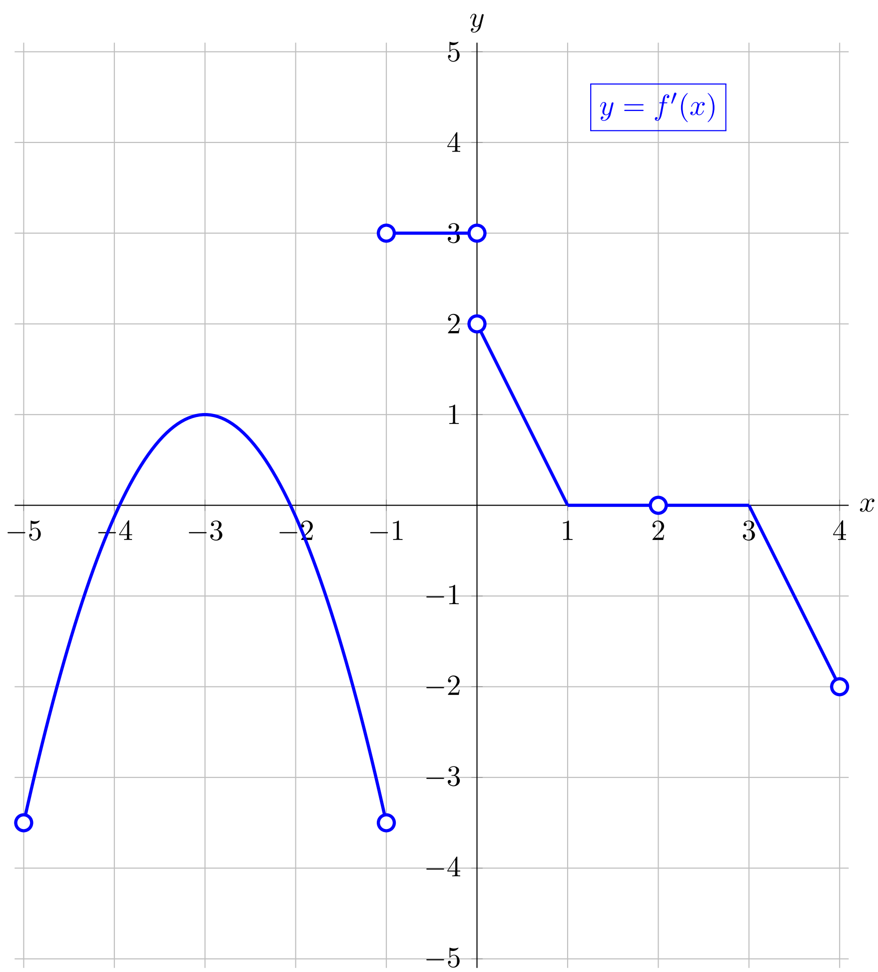
iv. When a sheep produces 12 pounds of wool per year, feeding it an extra pound of food per day will cause it to produce about 0.7 additional pounds of wool per year.

4. [8 points] The graph of a function f is shown below.



On the axes below, sketch a graph of $f'(x)$ (the derivative of the function $f(x)$) on the interval $-5 < x < 4$. Be sure that you pay close attention to each of the following:

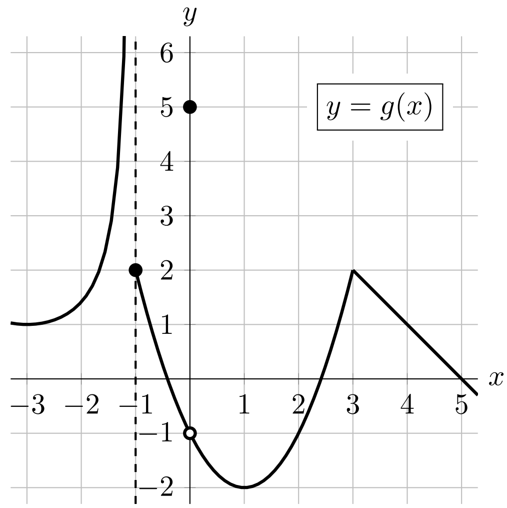
- where f' is defined
- the value of $f'(x)$ near each of $x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4$
- the sign of f'
- where f' is increasing/decreasing/constant



1. [10 points]
A portion of the graph of a function $g(x)$ is shown to the right, along with some values of an invertible, differentiable function $h(x)$ and its derivative $h'(x)$ below. Note that:

- $g(x)$ is linear on $[3, 5]$;
- $g(x)$ has a vertical asymptote at $x = -1$.

x	-2	0	2	4	6
$h(x)$	-1	$-e^{-1}$	0	$\sqrt{2}$	e
$h'(x)$	2	1	π	5	$\sqrt{3}$



a. [2 points] Let $M(x) = x^2 h(x)$. Find $M'(-2)$.

Solution: By the Product Rule, $M'(x) = 2xh(x) + x^2h'(x)$, so

$$M'(-2) = -4h(-2) + 4h'(-2) = (-4)(-1) + 4(2) = 12.$$

Answer: $M'(-2) =$ 12

b. [2 points] Let $K(x) = \dfrac{g(x)}{h(x)}$. Find $K'(4)$.

Solution: Using the Quotient Rule,

$$K'(4) = \frac{g'(4)h(4) - g(4)h'(4)}{h(4)^2} = \frac{(-1)(\sqrt{2}) - (1)(5)}{2} = \frac{-\sqrt{2}-5}{2}.$$

Answer: $K'(4) = \dfrac{-\sqrt{2}-5}{2}$

c. [2 points] Find $(h^{-1})'(0)$.

Solution: Using the rule for the derivative of an inverse function,

$$(h^{-1})'(0) = \frac{1}{h'\big(h^{-1}(0)\big)} = \frac{1}{h'(2)} = \frac{1}{\pi}.$$

Answer: $(h^{-1})'(0) = \dfrac{1}{\pi}$

d. [2 points] On which of the following intervals does $g(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle all correct answers.

$[-3, -1]$ $\qquad$ $[0, 2]$ $\qquad$ $\boxed{[1, 3]}$ $\qquad$ $[2, 4]$ $\qquad$ NONE OF THESE

e. [2 points] On which of the following intervals does $g(x)$ satisfy the conclusion of the Mean Value Theorem? Circle all correct answers.

$\boxed{[-3, -1]}$ $\qquad$ $[0, 2]$ $\qquad$ $\boxed{[1, 3]}$ $\qquad$ $[2, 5]$ $\qquad$ NONE OF THESE

3. [5 points] Let
$$K(u) = \arctan(u^2 + 3u).$$

Use the limit definition of the derivative to write an explicit expression for $K'(2)$. *Your answer should not involve the letter K. Do not attempt to evaluate or simplify the limit.* Write your final answer in the answer box provided below.

Solution:

$$K'(2) = \lim_{h\to 0} \frac{K(2+h) - K(2)}{h} = \lim_{h\to 0} \frac{\arctan\left((2+h)^2 + 3(2+h)\right) - \arctan(2^2 + 3\cdot 2)}{h}$$
$$= \lim_{h\to 0} \frac{\arctan\left(h^2 + 7h + 10\right) - \arctan(10)}{h}.$$

Answer: $K'(2) = \boxed{\lim_{h\to 0} \frac{\arctan\left((2+h)^2 + 3(2+h)\right) - \arctan(2^2 + 3\cdot 2)}{h}}$

4. [6 points] Suppose $b(x)$ is a differentiable function whose tangent line at the point $x = 4$ is given by the linear function $T(x)$. To the right is a table consisting of some values of $b(x)$ and $b'(x)$.

x	-3	-2	0	4
$b(x)$	5	1	-3	-6
$b'(x)$	-4	?	?	-1
$T(x)$	1	0	-2	-6

a. [2 points] Find the values of $T(x)$ at $x = -3, -2, 0,$ and 4, and write them into the table.

Solution: We use the fact that $T(x) = b(4) + b'(4)(x-4) = -6 - (x-4) = -2 - x$.

b. [2 points] Use the table to estimate $b'(-1)$.

Solution: $b'(-1) \approx \dfrac{b(0) - b(-2)}{0 - (-2)} = \dfrac{-3-1}{2} = \dfrac{-4}{2} = -2.$

Answer: -2

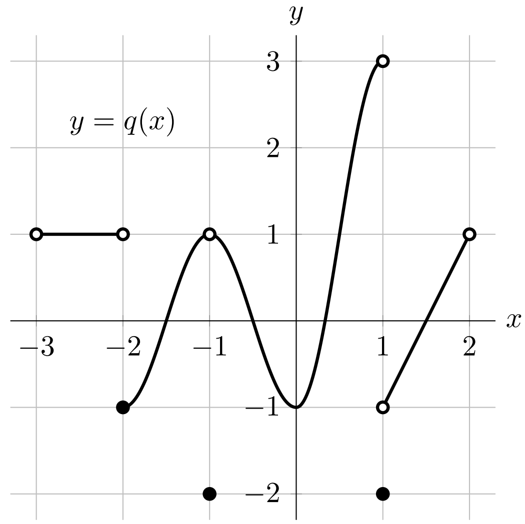
c. [2 points] Find an equation for the line tangent to the graph of $y = b(x)$ at the point $(-3, 5)$.

Solution: This line has equation

$$y = b(-3) + b'(-3)(x - (-3)) = 5 - 4(x+3) = -4x - 7.$$

Answer: $y = 5 - 4(x+3)$

3. [10 points] The entire graph of a function q is shown below. Note that $q(x)$ is linear on the interval $1 < x < 2$.



Throughout this problem, you do not need to explain your reasoning.

For each of parts **a.**– **c.** below, circle <u>all</u> of the listed values satisfying the given statement. If there are no such values, circle NONE.

a. [2 points] For which of the following values of a does $\lim_{t\to a} q(t)$ exist?

$a = -2$ $\boxed{a = -1}$ $\boxed{a = 0}$ $a = 1$ NONE

b. [2 points] For which of the following values of b is $q(x)$ continuous at $x = b$?

$b = -2$ $b = -1$ $\boxed{b = 0}$ $b = 1$ NONE

c. [2 points] For which of the following values of c is $\lim_{x\to c^+} q(x) = q(c)$?

$\boxed{c = -2}$ $c = -1$ $\boxed{c = 0}$ $c = 1$ NONE

For each of parts **d.** and **e.** below, if the limit does not exist (including the case of limits that diverge to ∞ or $-\infty$), write DNE.

d. [2 points] Evaluate the following expression: $\lim_{k\to 0} \dfrac{q(1.21 + k) - q(1.21)}{k}$.

Answer: 2

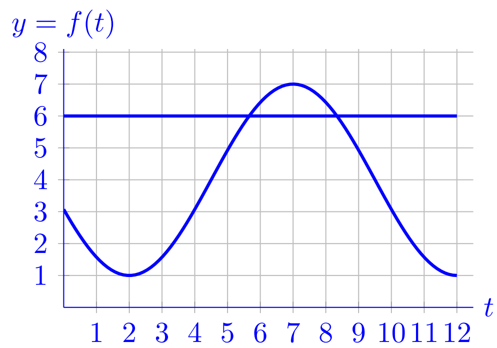
e. [2 points] Evaluate the following expression: $\lim_{s\to -1} q(q(s))$.

Answer: 3

7. [10 points] An apple farmer wants to assess the damage done by a plague to the trees in his orchard. In order to do so, he installs cameras on a couple of small flying robots to film the damage done by the plague to the trees. Let $f(t)$ and $s(t)$ and be the height above the ground (in feet) of the first and second robot t seconds after they started recording.

a. [5 points] Let $f(t) = 4 - 3\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right)$. Find the time(s) at which the first robot is 6 feet above the ground for $0 \le t \le 12$. Your answer(s) should be *exact*. Show all your work.

Solution: From the graph:



we see that there are two solutions.

$$4 - 3\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right) = 6$$

$$\cos\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right) = -\frac{2}{3}$$

$$\frac{\pi}{5}t - \frac{2\pi}{5} = \cos^{-1}\left(-\frac{2}{3}\right)$$

$$t = \frac{5}{\pi}\left(\cos^{-1}\left(-\frac{2}{3}\right) + \frac{2\pi}{5}\right)$$

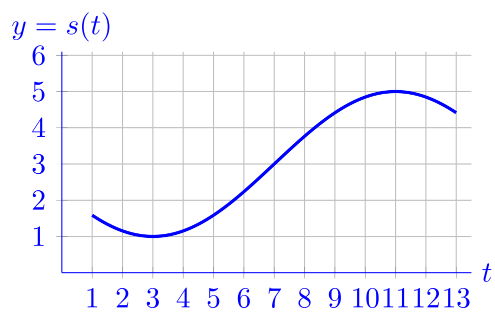
$$\frac{\pi}{5}t - \frac{2\pi}{5} = 2\pi - \cos^{-1}\left(-\frac{2}{3}\right)$$

$$t = 2 + \frac{5}{\pi}\left(2\pi - \cos^{-1}\left(-\frac{2}{3}\right)\right)$$

$$t = \frac{5}{\pi}\left(\cos^{-1}\left(-\frac{2}{3}\right) + \frac{2\pi}{5}\right), 2 + \frac{5}{\pi}\left(2\pi - \cos^{-1}\left(-\frac{2}{3}\right)\right)$$

b. [5 points] The graph of the sinusoidal function $s(t)$ is shown below only for $1 \le t \le 13$. Find a formula for $s(t)$.

Solution:

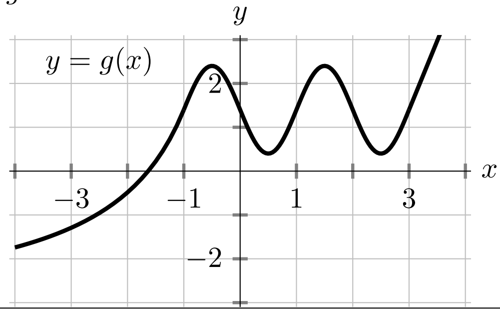


The sinusoidal $s(t) = -A\cos(B(t-h)) + k$ has:

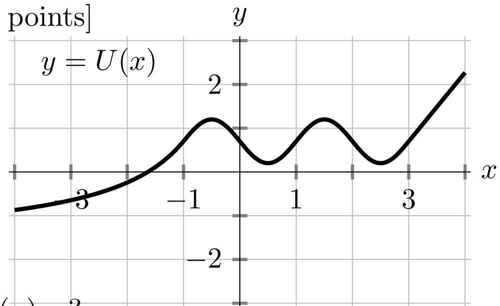
- Amplitude $= 2$ then $A = 2$.
- Midline $y = 3$ then $k = 3$.
- Period $= 16$ then $B = \frac{2\pi}{16} = \frac{\pi}{8}$.
- Horizontal shift $= 3$ then $h = 3$.

Hence $s(t) = -2\cos\left(\frac{\pi}{8}(t-3)\right) + 3$ (other formulas are also possible).

11. [11 points] A portion of the graph of a function g is shown below.
In each of parts **a.–d.** on this page, the corresponding portion of the graph of a function obtained from g by one or more transformations is shown, together with a list of possible formulas for that function. In each case, circle the one correct formula for the function shown.



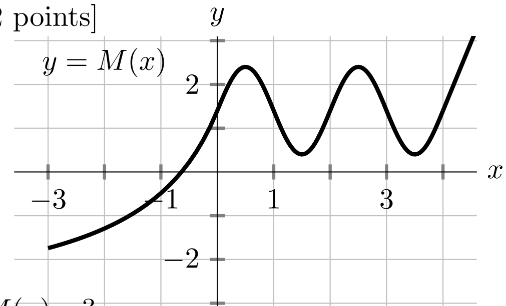
a. [2 points]



$U(x) =?$
Circle the one correct choice below.

$g(x) - 1$	$g(0.5x)$	$\boxed{0.5g(x)}$
$g(x) + 1$	$g(2x)$	$2g(x)$
$g(x) - 1.5$	$g(x+1)$	$g(x-1)$

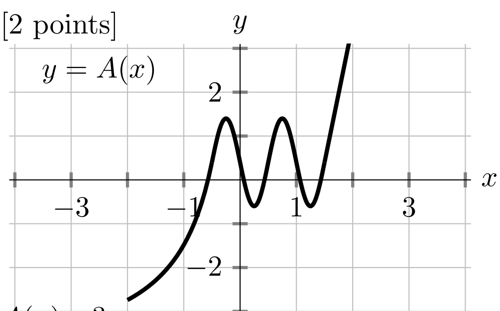
b. [2 points]



$M(x) =?$
Circle the one correct choice below.

$g(x) - 1$	$g(0.5x)$	$0.5g(x)$
$g(x) + 1$	$g(2x)$	$2g(x)$
$g(x) - 1.5$	$g(x+1)$	$\boxed{g(x-1)}$

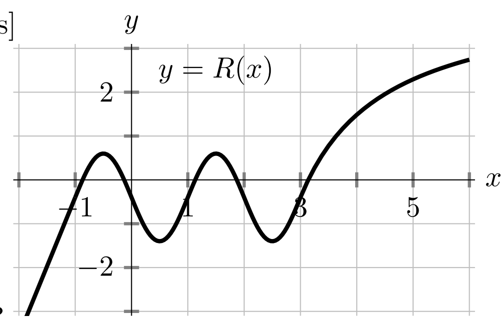
c. [2 points]



$A(x) =?$
Circle the one correct choice below.

$g(2x) + 1$	$g(0.5x) + 1$	$g(x-2) - 1$
$\boxed{g(2x) - 1}$	$g(0.5x) - 1$	$2g(x-1)$
$2g(x+1)$	$0.5g(x+1)$	$0.5g(x-1)$

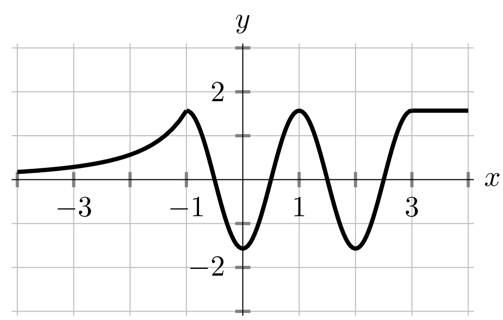
d. [2 points]



$R(x) =?$
Circle the one correct choice below.

$g(-x-1) + 2$	$-g(x-1) - 2$	$-g(x+2) - 1$
$g(-x+1) - 2$	$-g(-x-2) - 1$	$-g(x-2) + 1$
$g(-x-2) + 1$	$\boxed{-g(-x+2) + 1}$	$-g(-x+1) + 2$

e. [3 points] A portion of the graph of the derivative of one of the five functions above is shown on the right. Which derivative is shown? Circle the one correct choice below.



$g'(x)$ $\boxed{U'(x)}$ $M'(x)$ $A'(x)$ $R'(x)$

5. [12 points] A function g defined for all real numbers has the following properties:

(a) g is differentiable for $-1 \leq x < 4$.

(b) $g'(x) \leq 0$ for $-1 \leq x < 4$.

(c) $g''(x) > 0$ for $2 < x < 4$.

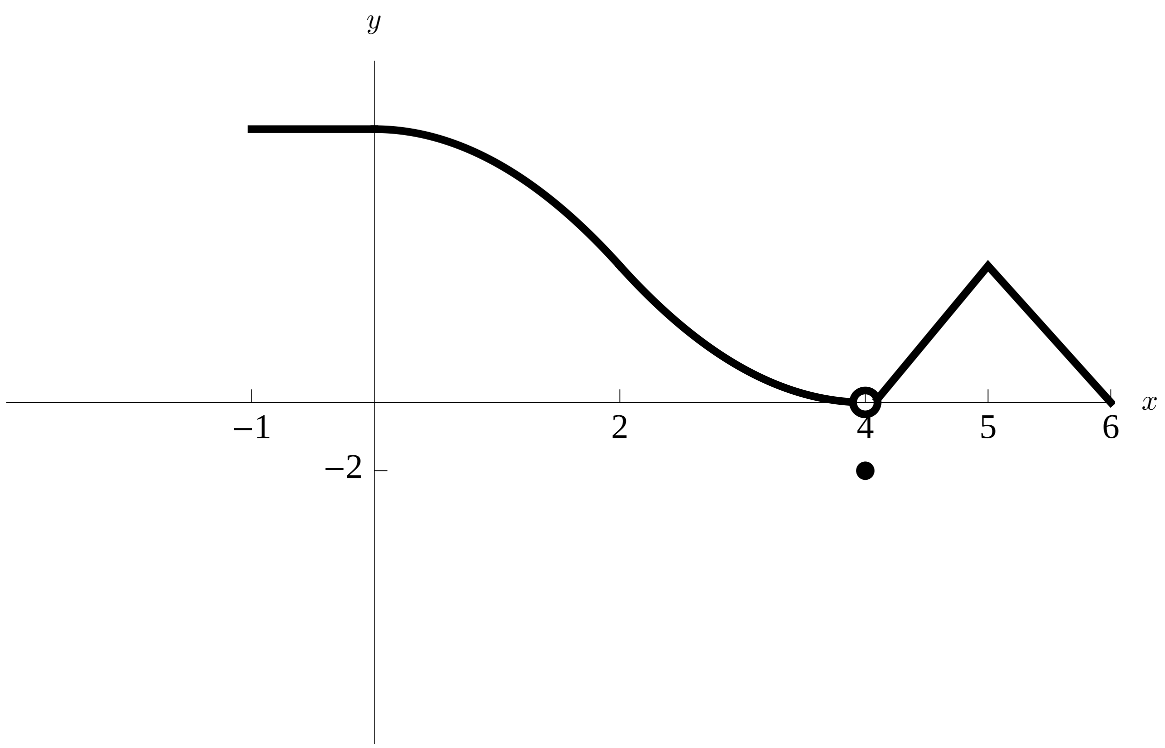
(d) $g(4) = -2$.

(e) $\lim_{x \to 4} g(x) = 0$.

(f) g is continuous at $x = 5$ but not differentiable at $x = 5$.

(g) $g'(0) = 0$.

On the axes below, draw a possible sketch of $y = g(x)$ on the domain $-1 \leq x \leq 6$, including labels.



4. [10 points] After the students planted the pine and the oak, the university has been monitoring the growth and health of the trees. Fifteen years after being planted, an invasion of cankerworms (a type of caterpillar) is found on the oak. It is predicted that the number of cankerworms (in hundreds) in the oak s weeks after the pest was detected is given by

$$C(s) = 2e^{0.35s}.$$

a. [2 points] By what percent is the population of cankerworms expected to grow every week?

Answer:
Since $b = e^{0.35}$, then $r = e^{0.35} - 1$. The population grows by $100r\% = 100(e^{0.35} - 1)\%$ every week.

Answer: The population grows by $100(e^{0.35} - 1)\% \approx 41.9\%$ every week

b. [3 points] Let $F(m)$ be the number of cankerworms in the oak (in <u>thousands</u>) m <u>days</u> after the pest was detected. Find a formula for $F(m)$ in terms of m only.

Answer: $F(m) = 0.2e^{0.05m}$

c. [5 points] A population of weevils (another insect) invades the pine. It is estimated that the population of weevils increases by 44 percent every 2 weeks. How many weeks does it take for the population of weevils to triple? *Show all your work and round your answer to the nearest week.*

Answer:
If the population of weevils is given by $W(t) = ab^t$, then the fact that it increases by 44 percent every 2 weeks yields $W(2) = 1.44W(0)$. In other words

$$ab^2 = 1.44a$$
$$b = \sqrt{1.44} = 1.2.$$

The time T it takes the population to triple satisfies $W(T) = 3W(0)$. Hence

$$a(1.2)^T = 3a \qquad (1.2)^T = 3.$$
$$T\ln(1.2) = \ln(3) \qquad T = \frac{\ln(3)}{\ln(1.2)}$$

Answer: $\dfrac{\ln(3)}{\ln(1.2)} \approx 6$ **weeks**