

MATH 115 — PRACTICE FOR EXAM 1

Generated October 25, 2026

UMID: SOLUTIONS INITIALS: _____

INSTRUCTOR: _____ SECTION NUMBER: _____

1. This exam has 13 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You may not use a calculator. You are allowed one double-sided 8×11 inch page of handwritten notes.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. You must use the methods learned in this course to solve all problems.
7. You are responsible for reading and following all directions provided for each exam in this course.

Semester	Exam	Problem	Name	Points	Score
Fall 2010	1	6	forecast	11	
Fall 2024	1	7	balloon	5	
Fall 2018	1	2		12	
Winter 2023	2	10		12	
Fall 2018	2	4		12	
Fall 2023	1	8	spaceship	8	
Fall 2022	2	3	water temp	8	
Winter 2017	2	8	apiary	8	
Fall 2013	1	2	bacon soda	10	
Fall 2013	1	3		5	
Fall 2022	2	4		4	
Fall 2022	2	5		7	
Fall 2016	1	4		10	
Total				112	

Recommended time (based on points): 109 minutes

6. [11 points] Aziza and Zainab are former Math 115 students at a prestigious weather forecasting company near Cloudytown, MI. Each using different meteorological instruments, they've recorded the rainfall over Cloudytown during a storm. They've let $F(t)$ be the total rainfall, in inches, t hours after the start of the storm. They collected the following data.

Aziza's data: $F(0) = 0$, $F(1) = 0.3$, and $F(2) = 0.5$.

Zainab's data: $F'(0) = 0.6$, $F'(1) = 0.7$, and $F'(2) = 0.3$.

- a. [4 points] Use Aziza's data (and *not* Zainab's data) to estimate how quickly the rain was falling, in inches per hour, at the start of the storm (time $t = 0$) and after one hour ($t = 1$).

Solution: At the start of the storm, the rainfall rate is $F'(0)$ which can be approximated by the average rate of change of F between $t = 0$ and $t = 1$:

$$F'(0) \approx \frac{F(1) - F(0)}{1 - 0} = 0.3.$$

Similarly, the rainfall rate after one hour is approximated by

$$F'(1) \approx \frac{F(2) - F(1)}{2 - 1} = 0.2.$$

(NOTE: Several answers are acceptable for the second part.)

$$t = 0 : \underline{0.3 \text{ in/hr}} \quad t = 1 : \underline{0.2 \text{ in/hr}}$$

- b. [4 points] (True or False) Circle "T" (True) or "F" (False) for each of the statements below.

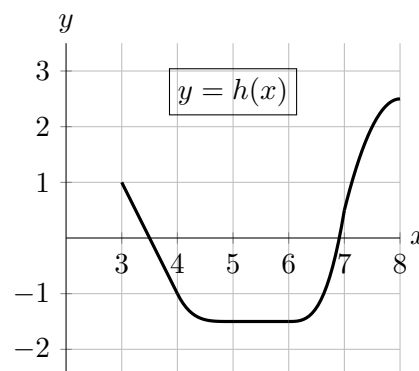
- Assuming all the data gathered was correct, throughout the second hour of the storm it was raining at a rate of about 0.7 inches per hour. F
- Assuming all the data gathered was correct, during the first hour of the storm rainfall slowed down and later sped up again. T
- Either Aziza's instrument or Zainab's instrument must be faulty because their measurements give different values for $F'(0)$ and $F'(1)$. F
- Assuming all the data gathered was correct, since $F'(0) = 0.6$ we know that about 0.6 inches of rain fell in the first hour. F

- c. [3 points] Give a practical interpretation of $F'(0) = 0.6$ that begins, "During the first five minutes of the storm..."

Solution: The equation $F'(0) = 0.6$ means the rainfall rate is 0.6 in/hr at the beginning of the storm. There are twelve five minute periods in an hour, so this rate is equivalent to 0.05 inches per 5 minutes. This means our interpretation should be "During the first five minutes of the storm, approximately 0.05 inches of rain fell."

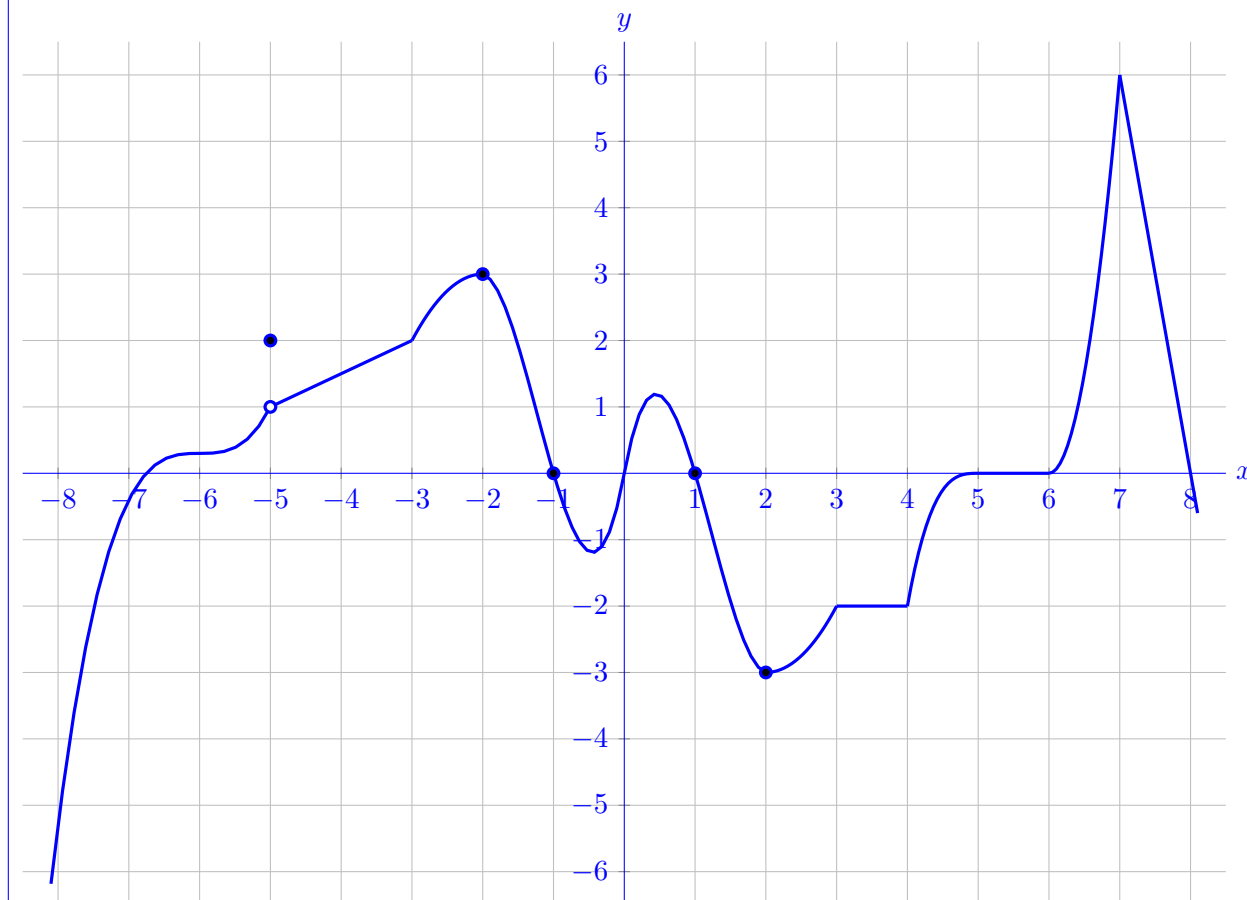
2. [12 points] On the axes provided below, sketch the graph of a single function $y = Q(x)$ satisfying all of the following conditions:

- The function $Q(x)$ is defined on $-8 \leq x \leq 8$.
- On the interval $(3, 8)$, the function $Q(x)$ is equal to the derivative of the function $h(x)$, which is shown in the graph at the right.
- $Q'(-6) = 0$ and $Q(x)$ is increasing in $-8 < x < -5$.
- $Q(x)$ is not continuous at $x = -5$ but $\lim_{x \rightarrow -5} Q(x)$ exists.
- $Q(-2) = 3$.
- $Q(x)$ has an x -intercept at $x = 1$.
- $Q(x) = -Q(-x)$ for $-3 < x < 3$.

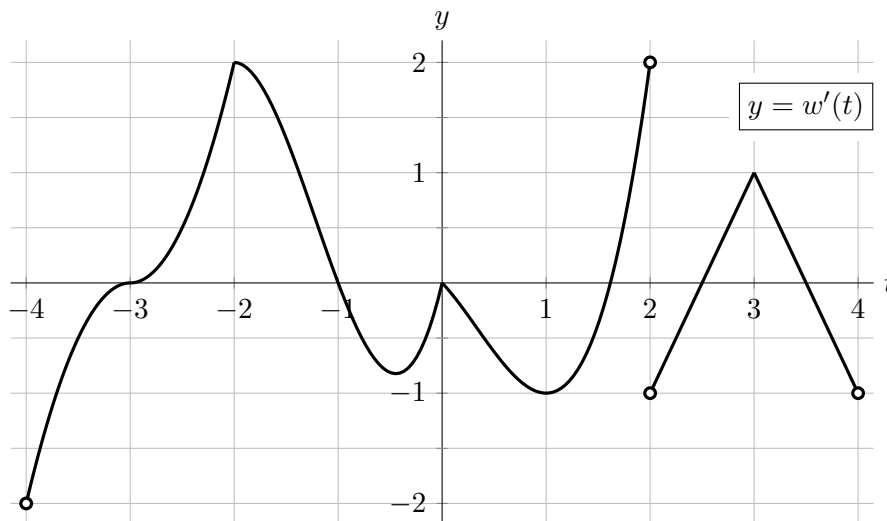


Make sure that your graph is large and unambiguous.

Solution:



10. [12 points] Suppose $w(t)$ is a continuous function, defined on the interval $(-4, 4)$. A graph of the derivative $w'(t)$ is given below.



- a. [2 points] Circle all points below that are critical points of $w(t)$.

☒ $t = -3$ ☐ $t = -2$ ☐ $t = 1$ ☒ $t = 2$ ☐ $t = 3$ ☐ NONE OF THESE

- b. [2 points] Circle all points below that are critical points of $w'(t)$.

☒ $t = -3$ ☒ $t = -2$ ☒ $t = 1$ ☐ $t = 2$ ☒ $t = 3$ ☐ NONE OF THESE

- c. [2 points] Circle all points below that are local minima of $w(t)$.

☒ $t = -3$ ☐ $t = -2$ ☐ $t = -1$ ☐ $t = 1$ ☐ $t = 2$ ☐ NONE OF THESE

- d. [2 points] Circle all points below that are local maxima of $w(t)$.

☐ $t = -3$ ☐ $t = -2$ ☒ $t = -1$ ☐ $t = 1$ ☒ $t = 2$ ☐ NONE OF THESE

- e. [2 points] Circle all points below that are inflection points of $w(t)$.

☐ $t = -3$ ☒ $t = -2$ ☒ $t = 1$ ☐ $t = 2$ ☒ $t = 3$ ☐ NONE OF THESE

- f. [1 point] Circle all points below that are global maxima of $w'(t)$ on the interval $(-4, 4)$.

☐ $t = -4$ ☒ $t = -2$ ☐ $t = 1$ ☐ $t = 2$ ☐ $t = 3$ ☐ NONE OF THESE

- g. [1 point] Circle all points below that are global minima of $w'(t)$ on the interval $(-4, 4)$.

☐ $t = -4$ ☐ $t = -2$ ☐ $t = 1$ ☐ $t = 2$ ☐ $t = 3$ ☒ NONE OF THESE

4. [12 points] Suppose $r(x)$ is a differentiable function defined for all real numbers x . The derivative and second derivative of $r(x)$ are given by

$$r'(x) = (x+1)^3(x-2)^{4/5} \quad \text{and} \quad r''(x) = \frac{(x+1)^2(19x-26)}{5(x-2)^{1/5}}$$

- a. [6 points] Find the x -coordinates of all critical points of $r(x)$ and all values of x at which $r(x)$ has a local extremum. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank below, write NONE if appropriate.

Solution: The critical points occur at values of x where $r'(x) = 0$ or $r'(x)$ does not exist. $r'(x)$ exists for all values of x , so the only critical points are at points where

$$r'(x) = (x+1)^3(x-2)^{4/5} = 0.$$

That is when $x = -1$ and $x = 2$.

Now we need to know the sign of $r'(x)$ on three intervals. We will consider the sign of each factor of x in order to do this.

0	sign of $(x+1)^3$	sign of $(x-2)^{4/5}$	sign of $r'(x)$
$-\infty < x < -1$	−	+	$(-)(+) = -$
$-1 < x < 2$	+	+	$(+)(+) = +$
$2 < x < \infty$	+	+	$(+)(+) = +$

This tells us that $r(x)$ is decreasing for $x < -1$ and increasing for $x > -1$. Therefore we have a local minimum at $x = -1$ and no local maxes.

Answer: Critical points at $x = -1, 2$

Local max(es) at $x = \text{NONE}$ Local min(s) at $x = -1$

- b. [6 points] Find the x -coordinates of all inflection points of $r(x)$. If there are none, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

Solution: The only potential inflection points occur when $r''(x) = 0$ or $r''(x)$ does not exist. From the formula, we see that $r''(x) = 0$ when $x = -1$ and $x = \frac{26}{19}$, and $r''(x)$ does not exist at $x = 2$.

Again, we need to make a table of the sign of $r''(x)$ on intervals between these points, because inflection points only occur when $r(x)$ changes concavity, which happens when $r''(x)$ changes sign.

0	sign of $(x+1)^2$	sign of $(19x-26)$	sign of $(x-2)^{1/5}$	sign of $r''(x)$
$-\infty < x < -1$	+	−	−	+
$-1 < x < \frac{26}{19}$	+	−	−	+
$\frac{26}{19} < x < 2$	+	+	−	−
$2 < x < \infty$	+	+	+	+

Therefore, $r''(x)$ changes concavity at $x = \frac{26}{19}$ and $x = 2$.

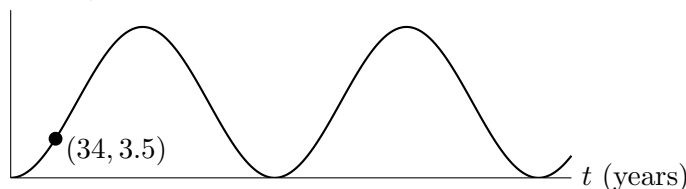
Answer: Inflection points at $x = \frac{26}{19}, 2$

8. [8 points] Far in the future, a particular spaceship transports humans back and forth along a fixed path between the two star systems Alpha and Beta. The distance d in light-years from Alpha to the spaceship t years after its initial departure from Alpha is given by

$$d = s(t) = 6.75 - 6.75 \cos\left(\frac{2\pi}{200}t\right).$$

A graph of $s(t)$, along with a table of a few values of both $s(t)$ and $s'(t)$, is given below.

d (light-years)



t	0	9	12	34
$s(t)$	0	0.27	0.47	3.50
$s'(t)$	0	0.06	??	0.19

- a. [1 point] How many years does it take for the spaceship to travel from Alpha to Beta?

Answer: 100 years.

- b. [1 point] How many light-years apart are Alpha and Beta?

Answer: 13.5 light-years.

- c. [1 point] Using the table, give the best possible estimate of $s'(12)$.

Answer: $\frac{0.47 - 0.27}{12 - 9}$ light-years per year.

There is a “Mystery Spot” along the spaceship’s path where gravity seems to be a little different. The ship first passes this spot 34 years after departing Alpha. (See the graph above.)

- d. [3 points] A baby tortoise is born on the spaceship just as it departs Beta, and becomes the ship’s mascot, remaining aboard for the rest of its life. Assuming the tortoise lives 212 years, how many times will it get to see the Mystery Spot, and at what age(s)?

Solution: Using graph symmetry and the fact that the period of $s(t)$ is 200, the next three times after $t = 34$ that the ship passes by the Mystery Spot are $t = 166$, $t = 234$, and $t = 366$. Since the ship is at Beta at time $t = 100$, this means the ship passes by the Mystery Spot 66 years, 134 years, and 266 years after being at Beta. But the tortoise only lives 212 years, so it will only see the Mystery Spot twice, at ages 66 and 134.

Answer: two times, at ages 66 and 134.

- e. [2 points] Now suppose the Mystery Spot wrecks the ship’s engines at time $t = 34$, so the spaceship is left to drift along forever at the speed and in direction it was going when it reached the Mystery Spot. Under this new assumption, how far away would the spaceship be from Alpha two years after it reached the Mystery Spot? (Include units. Note that $t = 34$ appears in the graph and in the table.)

Solution: At time $t = 34$, the spaceship is 3.5 light-years from Alpha and moving directly away from Alpha at a speed of 0.19 light-years per year. So if it keeps moving away from Alpha at this same speed, in two years it will be $2 \cdot 0.19 = 0.38$ light-years farther away, so its total distance from Alpha will be $3.5 + 0.38 = 3.88$ light-years.

Answer: $3.50 + 2 \cdot 0.19 = 3.88$ light-years

3. [8 points] At a certain location in Lake Michigan, scientists are measuring water temperature. Let $W(d)$ be the temperature, in degrees Fahrenheit ($^{\circ}F$), of the water at a depth of d meters (m). Shown below is a table of values of $W(d)$ and its derivative $W'(d)$, which are both defined and differentiable for all $d \geq 0$.

d	10	18	20	36	78	95
$W(d)$	62	56	55	50	43	41
$W'(d)$	-1.25	-0.60	-0.45	-0.28	-0.15	-0.10

Assume that between each pair of consecutive values of d given in the table, each function $W(d)$ and $W'(d)$ is either always increasing or always decreasing. *Throughout this problem, you do not need to include units or simplify numerical values.*

- a. [1 point] Use the table to approximate the value of $W''(19)$.

Answer: $W''(19) \approx \frac{0.15}{2}$

- b. [2 points] Write a formula for the linear approximation $L(d)$ of $W(d)$ near $d = 95$.

Answer: $L(d) = 41 - 0.1(d - 95)$

- c. [1 point] Use your formula from part b. to approximate the water temperature, in $^{\circ}F$, of the water at a depth of 90 meters.

Answer: $41 - 0.1(90 - 95) = 41.5$

- d. [1 point] Is your estimate from part c. an overestimate, an underestimate, neither, or is there not enough information to decide? Circle your answer.

Circle One: OVERESTIMATE UNDERESTIMATE NEITHER NOT ENOUGH INFO

- e. [3 points] The scientists are taking measurements using an underwater drone. The depth d , in meters, of the drone after t minutes of taking measurements can be modeled by $d = 3\sqrt{t}$. Let $R(t) = W(3\sqrt{t})$ be the temperature in $^{\circ}F$ outside the drone t minutes into the measurements. Write a formula for the linear approximation $K(t)$ of $R(t)$ near $t = 36$.

Solution: We know $K(t) = R(36) + R'(36)(t - 36)$. Now, $R(36) = W(3\sqrt{36}) = W(18) = 56$. Also, $R'(t) = W'(3\sqrt{t}) \cdot \frac{3}{2\sqrt{t}}$ by the Chain Rule, so $R'(36) = W'(18) \cdot \frac{3}{12} = -\frac{0.6}{4}$.

Answer: $K(t) = 56 - \frac{0.6}{4}(t - 36)$

8. [8 points] At Happy Hives Bee Farm, the population of bees, in thousands, t months after the farm opens, can be modeled by $g(t)$, where

$$g(t) = \begin{cases} 20 + \frac{1}{3}e^{4-t} & \text{for } 0 \leq t \leq 4 \\ -\frac{1}{6}t^3 + \frac{9}{4}t^2 - 7t + 23 & \text{for } 4 < t \leq 8 \end{cases}$$

and

$$g'(t) = \begin{cases} -\frac{1}{3}e^{4-t} & \text{for } 0 < t < 4 \\ -0.5(t-2)(t-7) & \text{for } 4 < t < 8. \end{cases}$$

- a. [6 points] Find the values of t that minimize and maximize $g(t)$ on the interval $[0, 8]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema. For each answer blank, write NONE if appropriate.

Solution: In this case, you need to establish if the function is continuous before using the procedure listed above.

- Continuity of $g(t)$ on $[0, 8]$:

Since $\lim_{t \rightarrow 4^+} g(t) = \lim_{t \rightarrow 4^+} -\frac{1}{6}t^3 + \frac{9}{4}t^2 - 7t + 23 = \frac{61}{3}$ and $\lim_{t \rightarrow 4^-} g(t) = \frac{61}{3} = \lim_{t \rightarrow 4^-} g(t) = g(4)$, $g(t)$ is continuous at 4. Both pieces are continuous, so $g(t)$ is continuous on the interval $[0, 4]$.

- Critical points of $g(t)$: Using the formula for $g'(t)$

– $g'(t) = 0$ on $(0, 4)$: Since $e^{4-t} \neq 0$ for any value of t , then $g'(t) \neq 0$ on $(0, 4)$.

– $g'(t) = 0$ on $(4, 8)$: In this case $-0.5(t-2)(t-7) = 0$ if $t = 2, 7$. Hence the only solution in $(4, 8)$ is $t = 7$.

– $g'(t)$ is undefined on $(0, 8)$. Based on the formula for $g'(t)$, the only point where $g'(t)$ could be undefined is $t = 4$. In this case $g'(4)$ does not exist since:

$$* \lim_{h \rightarrow 0^-} \frac{g(4+h) - g(4)}{h} = -\frac{1}{3}e^{4-4} = -\frac{1}{3}$$

$$* \lim_{h \rightarrow 0^+} \frac{g(4+h) - g(4)}{h} = -0.5(4-2)(4-7) = 3$$

Hence the critical points of $g(t)$ in $(0, 8)$ are $t = 4, 7$.

Next we make a table to list $g(t)$ at all critical points and endpoints, and choose the values of t corresponding to the min and max from the table.

t	0	4	7	8
$g(t)$	38.199	20.33	27.08	25.67

Answer: Global max(es) at $t =$ 0

Answer: Global min(s) at $t =$ 4

- b. [2 points] What is the largest population of bees that occurs in the first 8 months the farm is open?

Answer: 38.199 thousand (or 38,199)

2. [10 points] Louis owns a small soda company and is experimenting with new flavors. Let $b(p)$ model the number of thousands of bottles of bacon-flavored soda sold by his company per month if he charges p cents per bottle. You may assume $b(p)$ is differentiable and invertible.

a. [2 points] Give a practical interpretation of the statement $b^{-1}(8) = 150$.

Solution: In order to sell 8000 bottles of bacon-flavored soda per month, the company should charge 150 cents per bottle.

b. [3 points] Give a practical interpretation of the statement $(b^{-1})'(4) = -10$.

Solution: In order to increase the number of bottles sold per month from 4000 to 5000, the company should lower the price about 10 cents.
If the company is currently selling 4000 bottles per month, lowering the price by 10 cents will increase sales by about 1000 bottles per month.
(There are other possible answers.)

c. [3 points] Write an expression that is equal to the price (in cents) that the company would have to charge per bottle in order to sell twice as many bottles of bacon-flavored soda as it sells at a price of 125 cents per bottle.

Solution: $b^{-1}(2b(125))$

d. [2 points] Which of the following is a correct formula for a function $h(d)$ that gives the number of thousands of bottles sold per month at a price of d dollars per bottle? (Circle your answer.)

$$h(d) = 100b(d) \qquad h(d) = \frac{b(d)}{100} \qquad \boxed{h(d) = b(100d)} \qquad h(d) = b\left(\frac{d}{100}\right)$$

3. [5 points] Use the limit definition of the derivative to write an explicit expression for $r'(3)$ where $r(t) = (t + 5)^{2t}$. Do not simplify or evaluate the limit. Your answer should not include the letter r .

Solution:

$$r'(3) = \lim_{h \rightarrow 0} \frac{(3 + h + 5)^{2(3+h)} - (3 + 5)^{2(3)}}{h}$$

2. [10 points] Louis owns a small soda company and is experimenting with new flavors. Let $b(p)$ model the number of thousands of bottles of bacon-flavored soda sold by his company per month if he charges p cents per bottle. You may assume $b(p)$ is differentiable and invertible.

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Solution: $b^{-1}(2b(125))$

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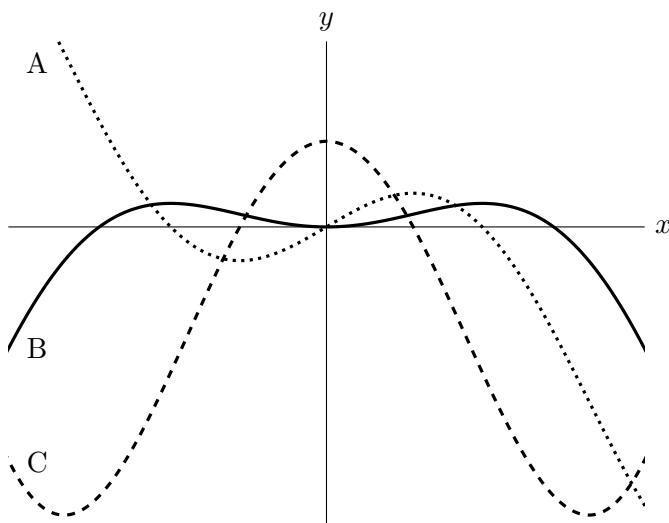
$$h(d) = 100b(d) \qquad h(d) = \frac{b(d)}{100} \qquad \boxed{h(d) = b(100d)} \qquad h(d) = b\left(\frac{d}{100}\right)$$

3. [5 points] Use the limit definition of the derivative to write an explicit expression for $r'(3)$ where $r(t) = (t + 5)^{2t}$. Do not simplify or evaluate the limit. Your answer should not include the letter r .

Solution:

$$r'(3) = \lim_{h \rightarrow 0} \frac{(3 + h + 5)^{2(3+h)} - (3 + 5)^{2(3)}}{h}$$

4. [4 points] Shown below are portions of the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



Answer: $f(x) :$ B

$f'(x) :$ A

$f''(x) :$ C

5. [7 points] The function $p(x)$ is given by the following formula, where c and d are nonzero constants:

$$p(x) = \begin{cases} \frac{1}{3}x^3 - 9x + 1 & x \leq 0 \\ 2^x & 0 < x < 2 \\ c + d(x - 2) & x \geq 2. \end{cases}$$

- a. [3 points] Find one pair of values for c and d such that $p(x)$ is differentiable at $x = 2$. Show your work.

Solution: For $p(x)$ to be differentiable at $x = 2$, it must first be continuous there, so we need $c + d(2 - 2) = c$ to equal $2^2 = 4$. Then, we need the slopes on either side of $x = 2$ to be the same. For $0 < x < 2$ we have $p'(x) = \ln(2)2^x$, while for $x > 2$ we have $p'(x) = d$, so we need $d = \ln(2)2^2$.

Answer: $c =$ 4 and $d =$ $4\ln(2)$

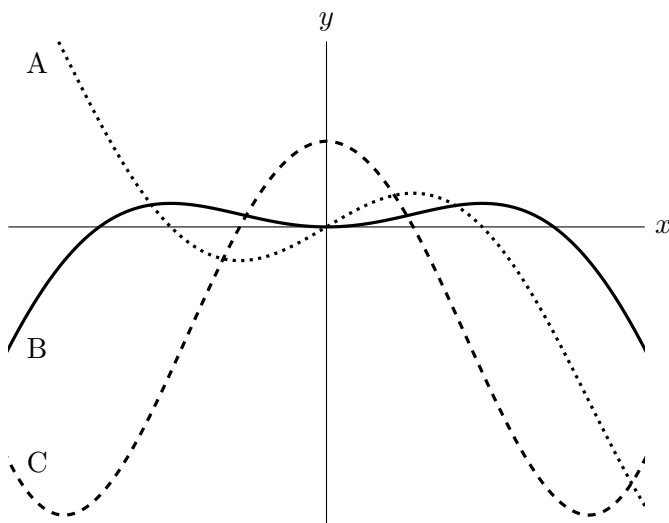
- b. [4 points] For the values of c and d from part a., find the x -coordinates of all critical points of $p(x)$ or write NONE if there are none. Show your work.

Solution: For $x < 0$ we have $p'(x) = x^2 - 9$. Then, as above, for $0 < x \leq 2$ we have $p'(x) = \ln(2)2^x$, while for $x \geq 2$ we have $p'(x) = d = 4\ln(2)$. Recall that we chose the values of c and d in part a. so that $p(x)$ was differentiable at $x = 2$, so we know $p'(2)$ exists. However, we need to consider whether $p'(x)$ exists at $x = 0$: to the left of $x = 0$, the slope is $0^2 - 9 = -9$, while to the right, the slope is $\ln(2)$, and since these are not equal, $p'(x)$ does not exist at $x = 0$, which is therefore a critical point of $p(x)$.

Then, we note that $p'(x)$ is never 0 for $x > 0$, but $x^2 - 9 = 0$ when $x = \pm 3$. However, this formula is only relevant for $x < 0$, so this only gives -3 as an additional critical point.

Answer: Critical point(s) at $x =$ $-3, 0$

4. [4 points] Shown below are portions of the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



Answer: $f(x) : \underline{\text{B}}$

$f'(x) : \underline{\text{A}}$

$f''(x) : \underline{\text{C}}$

5. [7 points] The function $p(x)$ is given by the following formula, where c and d are nonzero constants:

$$p(x) = \begin{cases} \frac{1}{3}x^3 - 9x + 1 & x \leq 0 \\ 2^x & 0 < x < 2 \\ c + d(x - 2) & x \geq 2. \end{cases}$$

- a. [3 points] Find one pair of values for c and d such that $p(x)$ is differentiable at $x = 2$. Show your work.

Solution: For $p(x)$ to be differentiable at $x = 2$, it must first be continuous there, so we need $c + d(2 - 2) = c$ to equal $2^2 = 4$. Then, we need the slopes on either side of $x = 2$ to be the same. For $0 < x < 2$ we have $p'(x) = \ln(2)2^x$, while for $x > 2$ we have $p'(x) = d$, so we need $d = \ln(2)2^2$.

Answer: $c = \underline{4}$ and $d = \underline{4\ln(2)}$

- b. [4 points] For the values of c and d from part a., find the x -coordinates of all critical points of $p(x)$ or write NONE if there are none. Show your work.

Solution: For $x < 0$ we have $p'(x) = x^2 - 9$. Then, as above, for $0 < x \leq 2$ we have $p'(x) = \ln(2)2^x$, while for $x \geq 2$ we have $p'(x) = d = 4\ln(2)$. Recall that we chose the values of c and d in part a. so that $p(x)$ was differentiable at $x = 2$, so we know $p'(2)$ exists. However, we need to consider whether $p'(x)$ exists at $x = 0$: to the left of $x = 0$, the slope is $0^2 - 9 = -9$, while to the right, the slope is $\ln(2)$, and since these are not equal, $p'(x)$ does not exist at $x = 0$, which is therefore a critical point of $p(x)$.

Then, we note that $p'(x)$ is never 0 for $x > 0$, but $x^2 - 9 = 0$ when $x = \pm 3$. However, this formula is only relevant for $x < 0$, so this only gives -3 as an additional critical point.

Answer: Critical point(s) at $x = \underline{-3, 0}$

4. [10 points] Consider the function f defined by $f(x) = \frac{(x + 1.8)(x + 2.1)}{(2x + 1.8)(3x - 6.9)(x + 2.1)}$.

You do not have to show your work/reasoning on this problem. However, any work that you do show may be considered for partial credit.

- a. [3 points] What is the domain of f ?

Answer: all real numbers except -0.9 , 2.3 , and -2.1

- b. [2 points] Find the equations of all vertical asymptotes of the graph of $y = f(x)$.
If there are none, write NONE.

Answer: $x = -0.9$ and $x = 2.3$

- c. [2 points] Let $g(x) = e^{-0.4x}$.

Find the equations of all horizontal asymptotes of the graph of $y = \frac{g(x)}{f(x)}$.

If there are none, write NONE.

Solution: $g(x)$ is a positive exponential decay function and dominates any rational function as $x \rightarrow \infty$. In particular, $\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0$ and $\lim_{x \rightarrow -\infty} \frac{g(x)}{f(x)} = \infty$ (DNE), so the only horizontal asymptote of the graph of $y = \frac{g(x)}{f(x)}$ is $y = 0$.

Answer: $y = 0$

- d. [3 points] Find a formula for a rational function $h(x)$ such that $\lim_{x \rightarrow \infty} \frac{f(x)}{h(x)} = 8$.

Solution: There are many possible answers. Some examples include:

- $h(x) = \frac{1}{8 \cdot 6 \cdot x} = \frac{1}{48x}$, and
- $h(x) = \frac{1}{8}f(x) = \frac{(x + 1.8)(x + 2.1)}{8(2x + 1.8)(3x - 6.9)(x + 2.1)}$.

Answer: $h(x) = \frac{1}{48x}$