Math 116 — Practice for Exam 3

Generated April 21, 2024

Name: ____

Instructor: ______________________ Section Number: __________

1. This exam has 11 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2019	2	5		9	
Fall 2018	3	10		6	
Winter 2022	3	8		11	
Fall 2023	3	1		12	
Winter 2017	2	6	Venice Beach	13	
Winter 2017	2	2	lemniscate	12	
Fall 2020	1	2	stork	11	
Fall 2023	3	5	beets	7	
Winter 2022	2	4	isosceles tank	15	
Fall 2017	1	2	catenary	13	
Fall 2009	2	3		12	
Total				121	

Recommended time (based on points): 120 minutes

5. [9 points]

a. [5 points] Determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n!(3n)}{(2n)!\,3^n}(x-7)^n.$$

Show all your work.

Radius: ____________________

b. [4 points] The power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6^n\sqrt{n^2+n+7}}(x-4)^n$ has radius of convergence $R=6$.

At which of the following x-values does the power series converge? Circle all correct answers. You do not need to justify your answer.

i. $x=-6$

ii. $x=-2$

iii. $x=0$

iv. $x=4$

v. $x=6$

vi. $x=10$

vii. $x=12$

viii. NONE OF THESE

10. [6 points] The Taylor series centered at $x = 0$ for a function $F(x)$ converges to $F(x)$ for $-e^{-1} < x < e^{-1}$ and is given below.

$$F(x) = \sum_{n=0}^{\infty} \frac{(n+1)^n}{n!} x^n \quad \text{for } -\frac{1}{e} < x < \frac{1}{e}.$$

a. [2 points] What is $F^{(2018)}(0)$? Make sure your answer is exact. You do not need to simplify.

Answer: $F^{(2018)}(0) =$ ____________________

b. [4 points] Use appropriate Taylor series for $F(x)$ and $\cos(x)$ to compute the following limit:

$$\lim_{x \to 0} \frac{(F(x) - 1)(\cos(x) - 1)}{x^3}$$

Show your work carefully.

Answer: $\lim_{x \to 0} \frac{(F(x) - 1)(\cos(x) - 1)}{x^3} =$ ____________________

8. [11 points]

a. [6 points] Give the first three non-zero terms of the Taylor Series for the function:

$$(x^2 + 2)\sin(x)$$

centered at $x = 0$.

b. [5 points] Compute the limit:

$$\lim_{x\to 0} \frac{\displaystyle\int_0^{2x} \left(\left(\frac{t}{2}\right)^2 + 2\right) \sin\left(\frac{t}{2}\right) dt}{x^2}$$

Answer: ____________________

1. [12 points] Compute the exact value of each of the following, if possible. Your answers should not involve integration signs, ellipses or sigma notation. For any values which do not exist, write **DNE**. You do not need to show work.

a. [2 points] The integral $\int_{-10}^{10} (f(x)+1)\ \mathrm{d}x$, where $f(x)$ is an odd function.

Answer: ______________________

b. [2 points] The integral $\int_{-3}^{4} \frac{1}{x^4}\ \mathrm{d}x$.

Answer: ______________________

c. [2 points] The sum $\sum_{n=0}^{2023} 7(5)^n$.

Answer: ______________________

d. [2 points] The **radius of convergence** for the Taylor series centered around $x = 0$ for the function $g(x) = (1 + 3x^2)^{1/5}$.

Answer: ______________________

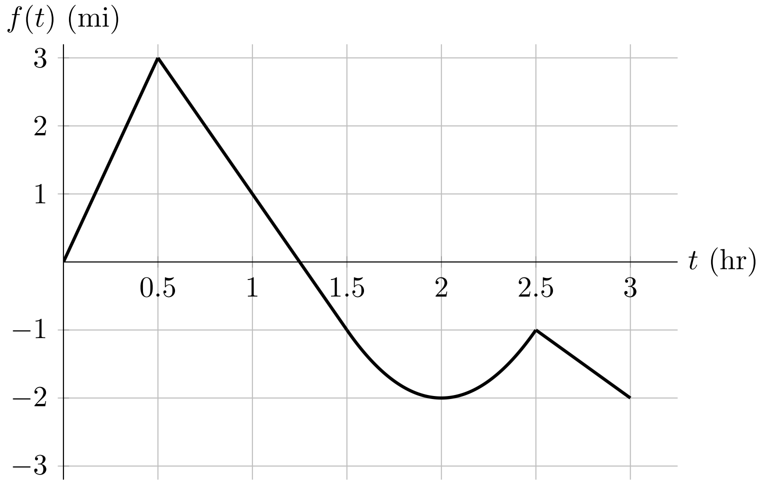
e. [2 points] The infinite sum $(0.5)^2 - \frac{(0.5)^4}{2} + \frac{(0.5)^6}{3} - \cdots + \frac{(-1)^{n+1}(0.5)^{2n}}{n} + \cdots$.

Answer: ______________________

f. [2 points] The value of $h''(2)$ where the fourth-degree Taylor polynomial for $h(x)$ about $x = 2$ is given by $P_4(x) = 2 + 9(x-2) - 81(x-2)^4$.

Answer: ______________________

6. [13 points] Anderson and Glen decide to take a road trip starting from Venice Beach. They have no worries about getting anywhere quickly, as they enjoy each other's company, so they take a very inefficient route. Suppose that Venice Beach is located at $(0, 0)$ and that Anderson and Glen's position (x, y) (measured in miles) t hours after leaving Venice Beach is given by a pair of parametric equations $x = f(t)$, $y = g(t)$. A graph of $f(t)$ and a formula for $g(t)$ are given below. Note that $f(t)$ is linear on the intervals $[0, 0.5]$, $[0.5, 1.5]$, and $[2.5, 3]$.



$$g(t) = -t^3 + 5t^2 - 3t$$

Note: For each of the following, your final answer should **not** involve the letters f and g.

a. [2 points] If their roadtrip last 3 hours, what are the x– and y– coordinates of their final destination?

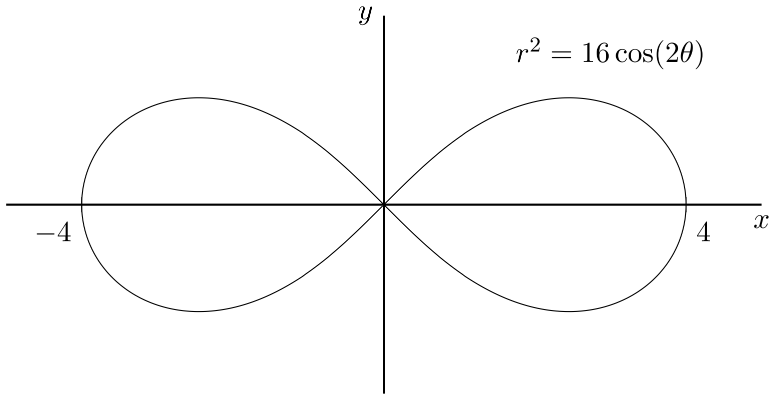
b. [3 points] At what speed are they traveling 2 hours into their trip?

c. [4 points] Write, but do not compute, an expression involving one or more integrals that gives the distance they traveled, in miles, in the first half hour of their trip.

d. [4 points] Write down a pair of parametric equations using the parameter s for the line tangent to their path at $t = 2.75$ hours.

Answer: $x(s) =$ ________________ and $y(s) =$ ________________

2. [12 points] Chancelor was doodling in his coloring book one Sunday afternoon when he drew an infinity symbol, or lemniscate. The picture he drew is the polar curve $r^2 = 16\cos(2\theta)$, which is shown on the axes below. (The axes are measured in inches.)

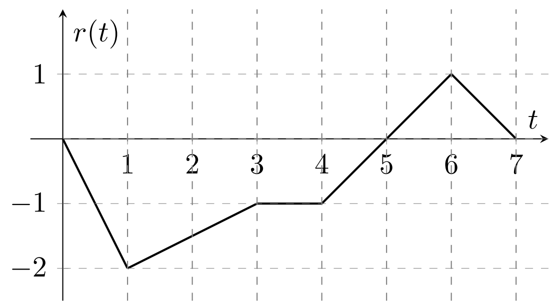


a. [4 points] Chancelor decides to color the inside of the lemniscate red. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total area, in square inches, that he has to fill in with red.

b. [4 points] He decides he wants to outline the right half (the portion to the right of the y-axis) of the lemniscate in blue. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total length, in inches, of the outline he must draw in blue.

c. [4 points] Chancelor draws another picture of the same lemniscate, but this time also draws a picture of the circle $r = 2\sqrt{2}$. He would like to color the area that is inside the lemniscate but outside the circle purple. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total area, in square inches, that he must fill in with purple.

2. [11 points] In the game of *Vegetable Crossing*, Tina is carefully monitoring the stork market, which determines the price of a stork in dubloons, the game's currency. If t is the number of days since Tina started playing, then $r(t)$, measured in dubloons per day, gives the **rate of change** of the price of a stork in the game. A graph of $r(t)$ is shown below. Note that $r(t)$ is piecewise linear.



a. [2 points] For what value of t in $[0, 7]$ is the price of a stork growing fastest?

b. [2 points] Tina wants to buy storks when the price is as low as possible. For what value of t in $[0, 7]$ should she buy storks?

c. [3 points] What is the average value of $r(t)$ on the interval $[3, 5]$? Be sure to write down any integrals you use to obtain your answer.

d. [4 points] Let $R(t)$ be the price of a stork in dubloons at time t, and assume that $R(t)$ is continuous. The price of a stork at time $t = 3$ is 14 dubloons. Given that information, fill out the following table of values:

t	0	2	4	6
$R(t)$				

5. [7 points] A local beet company, Dope Beets Inc., is developing a new beet with an adjustable growth rate for its many different customers. The growth rate of their new beet, measured in pounds per day, t days after a beet is planted, is given by

$$r(t) = \frac{5t^2}{t^k + t + 1},$$

for some adjustable value $k > 1$.

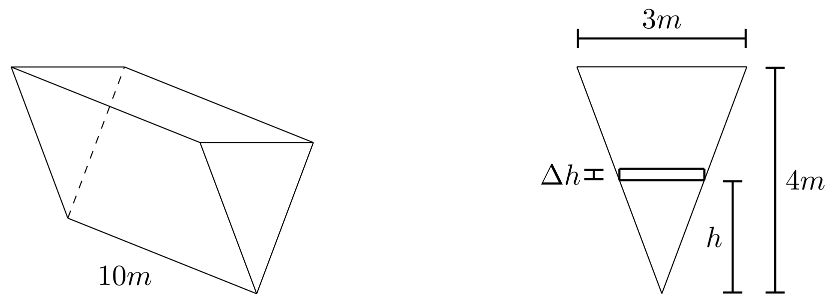
a. [4 points] Suppose a new beet initially weighs 2 pounds. Write an expression involving an integral for the weight, in pounds, of the beet t days after it is planted.

Answer: ______________________________

b. [3 points] Dope Beets Inc. wants to adjust the value of k such that a planted beet will never have infinite weight, even if the beet is allowed to grow forever. Which values of k would keep the weight finite? Give your answer as a value, list of values, or interval, as appropriate. No justification is required.

Answer: ______________________________

4. [15 points] A gas station needs to pump gas out of a subterranean tank. The tank is 10 meters in length, and has cross-sections shaped like isosceles triangles, with base 3 meters and height 4 meters. **The top of the tank is 15 meters below the surface of the earth**. Recall that $g = 9.8 m/s^2$ is the gravitational constant.



Underground Tank

a. [5 points] Write an expression for the volume (in cubic meters) of a horizontal rectangular slice of gasoline at height h above the bottom of the tank, with thickness Δh. Your answer should not involve an integral.

b. [3 points] Gasoline has a density of 800 kg/m^3. Write an expression for the weight (in newtons) of the slice of gasoline mentioned in part (a). Your answer should not involve an integral.

c. [4 points] Write an expression for the work (in joules) needed to pump the slice of gasoline mentioned above to the surface of the earth. Your answer should not involve an integral.

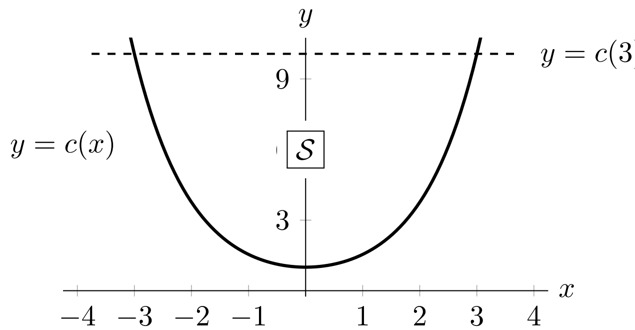
d. [3 points] Write an integral for the total work (in joules) needed pump all of the gasoline to the surface of the earth.

2. [13 points]

Consider the function c defined for all real numbers x by the formula

$$c(x) = \frac{e^x + e^{-x}}{2}.$$

A portion of the graph of this "catenary" function is shown as the solid curve in the graph on the right.



Let $\mathcal{S}$ be the region bounded by the graph of $y = c(x)$ and the line $y = c(3)$. This region $\mathcal{S}$ is shown in the figure above.

a. [2 points] Write, but do **not** evaluate, an expression involving one or more integrals that gives the area of $\mathcal{S}$.

b. [5 points] A solid is obtained by rotating the region $\mathcal{S}$ about the x-axis. Write, but do **not** evaluate, an expression involving one or more integrals that gives the volume of this solid.

c. [3 points] Write, but do **not** evaluate, an expression involving one or more integrals that gives the arc length of the graph of $y = c(x)$ over the interval $-3 \le x \le 3$. (Your answer should **not** involve any function names.)

d. [3 points] Will the midpoint rule with 2000 subdivisions give an underestimate or an overestimate of the value of $\int_{-3}^{0} c(x)\,dx$?

Circle your answer below. Then briefly explain your reasoning in the space on the right.

Circle one: Briefly explain your reasoning.

Underestimate

Overestimate

Neither (They are equal)

Cannot be determined

3. [12 points] Indicate whether you think each of the following series converges, diverges, or whether there is not enough information to determine convergence. You do not need to show your work for this page.

a. [3 points] Suppose $\sum_{n=1}^{\infty}(a_n + b_n)$ converges. Determine the convergence of $\sum_{n=1}^{\infty} a_n$.

CONVERGES DIVERGES CANNOT TELL

b. [3 points] Suppose $\sum_{n=1}^{\infty} a_n$ converges. Determine the convergence of $\sum_{n=1}^{\infty}(a_n + 4)$.

CONVERGES DIVERGES CANNOT TELL

c. [3 points] Suppose $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$. Determine the convergence of $\sum_{n=1}^{\infty}\frac{a_n}{n}$.

CONVERGES DIVERGES CANNOT TELL

d. [3 points] Determine the convergence of $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$.

CONVERGES DIVERGES CANNOT TELL