Math 116 — Practice for Exam 3

Generated April 21, 2024

Name: **SOLUTIONS**

Instructor: ______________________ Section Number: ____________

1. This exam has 11 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2019	2	5		9	
Fall 2018	3	10		6	
Winter 2022	3	8		11	
Fall 2023	3	1		12	
Winter 2017	2	6	Venice Beach	13	
Winter 2017	2	2	lemniscate	12	
Fall 2020	1	2	stork	11	
Fall 2023	3	5	beets	7	
Winter 2022	2	4	isosceles tank	15	
Fall 2017	1	2	catenary	13	
Fall 2009	2	3		12	
Total				121	

Recommended time (based on points): 120 minutes

5. [9 points]

a. [5 points] Determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n!(3n)}{(2n)!\,3^n}(x-7)^n.$$

Show all your work.

Solution: Let $a_n = \frac{n!(3n)}{(2n)!\,3^n}(x-7)^n$. Then we will find the radius of convergence by applying the ratio test to $\sum_{n=0}^{\infty} a_n$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} &= \lim_{n\to\infty} \frac{\frac{(n+1)!(3n+3)}{(2n+2)!3^{n+1}}|x-7|^{n+1}}{\frac{n!(3n)}{(2n)!3^n}|x-7|^n} \\
&= \lim_{n\to\infty} \frac{(n+1)!(3n+3)(2n)!3^n|x-7|^{n+1}}{(2n+2)!3^{n+1}n!(3n)|x-7|^n} \\
&= \lim_{n\to\infty} \frac{(n+1)(3n+3)}{(2n+2)(2n+1)3(3n)}|x-7| \\
&= 0
\end{aligned}$$

for all x, since the degree of the denominator is greater than the degree of the numerator. This means that $\sum_{n=0}^{\infty} a_n$ converges for all x, and the radius is infinity.

Radius: ∞

b. [4 points] The power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6^n\sqrt{n^2+n+7}}(x-4)^n$ has radius of convergence $R = 6$.

At which of the following x-values does the power series converge? Circle all correct answers. You do not need to justify your answer.

i. $x = -6$

ii. $x = -2$

iii. $\boxed{x = 0}$

iv. $\boxed{x = 4}$

v. $\boxed{x = 6}$

vi. $\boxed{x = 10}$

vii. $x = 12$

viii. NONE OF THESE

10. [6 points] The Taylor series centered at $x = 0$ for a function $F(x)$ converges to $F(x)$ for $-e^{-1} < x < e^{-1}$ and is given below.

$$F(x) = \sum_{n=0}^{\infty} \frac{(n+1)^n}{n!} x^n \quad \text{for } -\frac{1}{e} < x < \frac{1}{e}.$$

a. [2 points] What is $F^{(2018)}(0)$? Make sure your answer is exact. You do not need to simplify.

$$\frac{F^{(2018)}(0)}{2018!} = \text{coefficient of } x^{2018} = \frac{(2018+1)^{2018}}{(2018)!}$$

Answer: $F^{(2018)}(0) =$ 2019^{2018}

b. [4 points] Use appropriate Taylor series for $F(x)$ and $\cos(x)$ to compute the following limit:

$$\lim_{x \to 0} \frac{(F(x)-1)(\cos(x)-1)}{x^3}$$

Show your work carefully.

$$F(x) = \frac{(0+1)^0}{0!} x^0 + \frac{(1+1)^1}{1!} x^1 + \frac{(2+1)^2}{2!} x^2 + \dots$$

$$= 1 + 2x + \frac{9}{2} x^2 + \dots$$

So $F(x) - 1 = 2x + \frac{9}{2} x^2 + \dots$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

So $\cos x - 1 = -\frac{x^2}{2} + \frac{x^4}{24} - \dots$

so we have $\displaystyle\lim_{x \to 0} \frac{(2x + \frac{9}{2}x^2 + \dots)(-\frac{x^2}{2} + \frac{x^4}{24} - \dots)}{x^3} = \lim_{x \to 0} \frac{-x^3 + (\text{powers of } x \text{ bigger than } 3)}{x^3}$

Answer: $\displaystyle\lim_{x \to 0} \frac{(F(x)-1)(\cos(x)-1)}{x^3} =$ -1

8. [11 points]

a. [6 points] Give the first three non-zero terms of the Taylor Series for the function:

$$(x^2+2)\sin(x)$$

centered at $x = 0$.

Solution: The function (x^2+2) is a polynomial, and therefore its own Taylor series. Next, we take the known Taylor series for $\sin(x)$. Since we need the first three non-zero terms, we start with the first 3 non-zero terms of $\sin(x)$. This gives:

$$(x^2+2)\sin(x) \approx (x^2+2)\left(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5\right)$$

If we expand the right side, we get:

$$x^3 - \frac{1}{3!}x^5 + \frac{1}{5!}x^7 + 2x - \frac{2}{3!}x^3 + \frac{2}{5!}x^5$$

Now, if we group terms, we get:

$$2x + (1 - \frac{2}{3!})x^3 + (\frac{2}{5!} - \frac{2}{3!})x^5 + \frac{1}{5!}x^7$$

Nothing has cancelled out, meaning the first three terms here are the first three non-zero terms we need. Therefore the final answer is:

$$2x + \left(1 - \frac{2}{3!}\right)x^3 + \left(\frac{2}{5!} - \frac{2}{3!}\right)x^5$$

b. [5 points] Compute the limit:

$$\lim_{x\to 0} \frac{\displaystyle\int_0^{2x} \left(\left(\frac{t}{2}\right)^2 + 2\right) \sin\left(\frac{t}{2}\right) dt}{x^2}$$

Solution: Solutions 1 and 2): If we examine the bounds of the integral, we see that as $x \to 0$, the numerator becomes zero. This means that the answer is in indeterminant form, and we can apply L'Hopital. Using second FTC, and chain rule, we get that the limit becomes:

$$\lim_{x\to 0} \frac{2((x^2+2)\sin(x))}{2x} = \lim_{x\to 0} \frac{(x^2+2)\sin(x)}{x}$$

From here, the limit becomes $\frac{0}{0}$ again, so we have two options. We can apply L'Hopital again, turning the limit into:

$$\lim_{x\to 0} \frac{(2x\sin(x) + (x^2+2)\cos(x))}{1} = \frac{(0 + (0^2+2)\cos(0))}{1} = 2$$

Or, instead of L'Hopital, we can plug in the answer from a) to get:

$$\lim_{x\to 0} \frac{2x + \left(1 - \frac{2}{3!}\right)x^3 + \left(\frac{2}{5!} - \frac{2}{3!}\right)x^5}{x} = \lim_{x\to 0} 2 + \left(1 - \frac{2}{3!}\right)x^2 + \left(\frac{2}{5!} - \frac{2}{3!}\right)x^4 = 2+0+0 = 2.$$

Solution 3): It is also possible to integrate the answer from a). Doing a u-sub, with $u = t/2$, before taking the limit, we get

$$\lim_{x\to\infty} \frac{2\int_0^x (u^2+2)\sin(u)du}{x^2}$$

Subbing in the answer from a) we get:

$$\lim_{x\to\infty} \frac{2\int_0^x 2u + \left(1 - \frac{2}{3!}\right)u^3 + \left(\frac{2}{5!} - \frac{2}{3!}\right)u^5)du}{x^2}$$

This becomes:

$$\lim_{x\to\infty} \frac{2(x^2 + \frac{1}{4}\left(1 - \frac{2}{3!}\right)x^4 + \frac{1}{6}\left(\frac{2}{5!} - \frac{2}{3!}\right)x^6)}{x^2}$$

Which becomes:

$$\lim_{x\to\infty} 2(1 + \frac{1}{4}\left(1 - \frac{2}{3!}\right)x^2 + \frac{1}{6}\left(\frac{2}{5!} - \frac{2}{3!}\right)x^4) = 2(1+0+0) = 2$$

Answer: 2

1. [12 points] Compute the exact value of each of the following, if possible. Your answers should not involve integration signs, ellipses or sigma notation. For any values which do not exist, write **DNE**. You do not need to show work.

a. [2 points] The integral $\int_{-10}^{10} (f(x)+1)\ \mathrm{d}x$, where $f(x)$ is an odd function.

Answer: 20

b. [2 points] The integral $\int_{-3}^{4} \frac{1}{x^4}\ \mathrm{d}x$.

Answer: **DNE**

c. [2 points] The sum $\sum_{n=0}^{2023} 7(5)^n$.

Answer: $\frac{7(1-5^{2024})}{1-5} = \frac{7}{4}(5^{2024}-1)$

d. [2 points] The **radius of convergence** for the Taylor series centered around $x=0$ for the function $g(x) = (1+3x^2)^{1/5}$.

Answer: $\frac{1}{\sqrt{3}}$

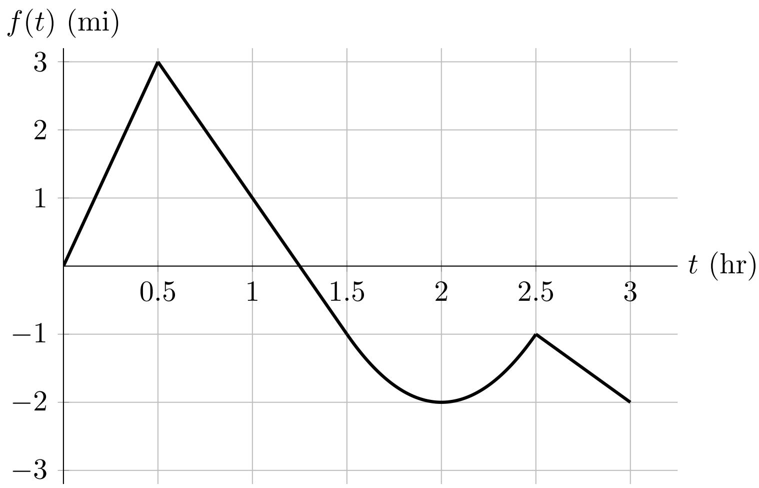
e. [2 points] The infinite sum $(0.5)^2 - \frac{(0.5)^4}{2} + \frac{(0.5)^6}{3} - \cdots + \frac{(-1)^{n+1}(0.5)^{2n}}{n} + \cdots$.

Answer: $\ln\left(\frac{5}{4}\right)$

f. [2 points] The value of $h''(2)$ where the fourth-degree Taylor polynomial for $h(x)$ about $x=2$ is given by $P_4(x) = 2 + 9(x-2) - 81(x-2)^4$.

Answer: **0**

6. [13 points] Anderson and Glen decide to take a road trip starting from Venice Beach. They have no worries about getting anywhere quickly, as they enjoy each other's company, so they take a very inefficient route. Suppose that Venice Beach is located at $(0,0)$ and that Anderson and Glen's position (x,y) (measured in miles) t hours after leaving Venice Beach is given by a pair of parametric equations $x = f(t)$, $y = g(t)$. A graph of $f(t)$ and a formula for $g(t)$ are given below. Note that $f(t)$ is linear on the intervals $[0, 0.5]$, $[0.5, 1.5]$, and $[2.5, 3]$.



$$g(t) = -t^3 + 5t^2 - 3t$$

Note: For each of the following, your final answer should **not** involve the letters f and g.

a. [2 points] If their roadtrip last 3 hours, what are the $x-$ and $y-$ coordinates of their final destination?

Solution: Note that at time $t = 3$, we have $x = f(3) = -2$ and $y = g(3) = 9$. So the coordinates of their final destination are $(-2, 9)$.

b. [3 points] At what speed are they traveling 2 hours into their trip?

Solution: We have $\left.\frac{dx}{dt}\right|_{t=2} = f'(2) = 0$ and $\left.\frac{dy}{dt}\right|_{t=2} = g'(2) = 5$.
So their speed at time $t = 2$ is $\sqrt{0^2 + 5^2} = 5$ miles per hour.

c. [4 points] Write, but do not compute, an expression involving one or more integrals that gives the distance they traveled, in miles, in the first **half** hour of their trip.

Solution: On the interval $(0, 0.5)$, we see that $f(t) = 6t$, so on this interval, we have

$$f'(t) = 6 \qquad \text{and} \qquad g'(t) = -3t^2 + 10t - 3.$$

The parametric arc length formula then implies that the distance they travelled from $t = 0$ to $t = 0.5$ is $\displaystyle\int_0^{0.5} \sqrt{(6)^2 + (-3t^2 + 10t - 3)^2}\, dt$ miles.

d. [4 points] Write down a pair of parametric equations using the parameter s for the line tangent to their path at $t = 2.75$ hours.

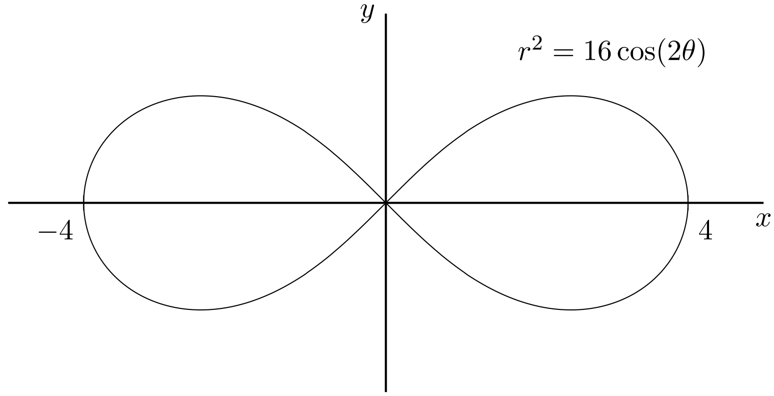
Solution: Note that

$$f(2.75) = -1.5, \quad \left.\frac{df}{dt}\right|_{t=2.75} = -2, \quad g(2.75) = 8.765625, \quad \text{and} \quad \left.\frac{dg}{dt}\right|_{t=2.75} = 1.8125$$

There are many possible parametrizations. There is no need to have this match with the parameter t from earlier, so the answer below has the line passing through $(-1.5, 8.765625)$ at $s = 0$.

Answer: $x(s) =$ $-2s - 1.5$ and $y(s) =$ $1.8125s + 8.765625$

2. [12 points] Chancelor was doodling in his coloring book one Sunday afternoon when he drew an infinity symbol, or lemniscate. The picture he drew is the polar curve $r^2 = 16\cos(2\theta)$, which is shown on the axes below. (The axes are measured in inches.)



a. [4 points] Chancelor decides to color the inside of the lemniscate red. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total area, in square inches, that he has to fill in with red.

Solution: Notice that we are given a formula for r^2 instead of r. Using symmetry, we will calculate the area in one quarter of the lemniscate and multiply by 4. To do this we will integrate from $\theta = 0$ to $\theta = \alpha$ where α is the smallest positive number for which $16\cos(2\alpha) = 0$. This gives $\alpha = \pi/4$. Using the formula for area inside a polar curve we see that the area is equal to $4 \cdot \frac{1}{2}\int_0^{\pi/4} 16\cos(2\theta)\, d\theta$ square inches.

b. [4 points] He decides he wants to outline the right half (the portion to the right of the y-axis) of the lemniscate in blue. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total length, in inches, of the outline he must draw in blue.

Solution: The portion of the lemniscate on the right of the y-axis corresponds to $-\pi/4 < \theta < \pi/4$. Notice that $\cos(2\theta) > 0$ for these angles.

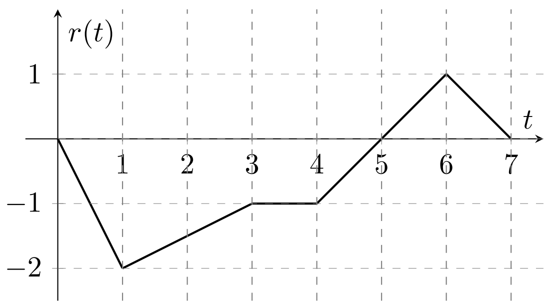
Implicitly differentiating $r^2 = 16\cos(2\theta)$ (or directly differentiating $r = 4(\cos(2\theta))^{1/2}$), we find that $\frac{dr}{d\theta} = \frac{-4\sin(2\theta)}{\sqrt{\cos(2\theta)}}$ so $\left(\frac{dr}{d\theta}\right)^2 = \frac{16\sin^2(2\theta)}{\cos(2\theta)}$.
Then using the arc length formula for polar coordinates we see that the length of the blue outline will be $\int_{-\pi/4}^{\pi/4} \sqrt{16\cos(2\theta) + \frac{16\sin^2(2\theta)}{\cos(2\theta)}}\, d\theta$ inches.

c. [4 points] Chancelor draws another picture of the same lemniscate, but this time also draws a picture of the circle $r = 2\sqrt{2}$. He would like to color the area that is inside the lemniscate but outside the circle purple. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total area, in square inches, that he must fill in with purple.

Solution: The circle and lemniscate intersect on the right side of the y-axis when $16\cos(2\theta) = 8$ or $\cos(2\theta) = \frac{1}{2}$. This gives angles $\theta_1 = -\pi/6$ and $\theta_2 = \pi/6$.

We first find the area between the curves on the right side using the formula for the area between polar curves and then multiply the by 2.
The resulting total area is $2 \cdot \frac{1}{2}\int_{-\pi/6}^{\pi/6} (16\cos(2\theta) - 8)\, d\theta$ square inches.

2. [11 points] In the game of *Vegetable Crossing*, Tina is carefully monitoring the stork market, which determines the price of a stork in dubloons, the game's currency. If t is the number of days since Tina started playing, then $r(t)$, measured in dubloons per day, gives the **rate of change** of the price of a stork in the game. A graph of $r(t)$ is shown below. Note that $r(t)$ is piecewise linear.



a. [2 points] For what value of t in $[0, 7]$ is the price of a stork growing fastest?

Solution: This will occur when $r(t)$ is at a maximum, so $t = 6$.

b. [2 points] Tina wants to buy storks when the price is as low as possible. For what value of t in $[0, 7]$ should she buy storks?

Solution: This will occur when the signed area between $r(t)$ and the t-axis is at a minimum, so $t = 5$.

c. [3 points] What is the average value of $r(t)$ on the interval $[3, 5]$? Be sure to write down any integrals you use to obtain your answer.

Solution: The average value of $r(t)$ on $[3, 5]$ is

$$\frac{1}{5-3}\int_3^5 r(t)dt.$$

Counting boxes using the grid, the integral has value -1.5, so the average value is -0.75.

d. [4 points] Let $R(t)$ be the price of a stork in dubloons at time t, and assume that $R(t)$ is continuous. The price of a stork at time $t = 3$ is 14 dubloons. Given that information, fill out the following table of values:

t	0	2	4	6
$R(t)$	18	$14 + \frac{5}{4}$	13	13

Solution: We get the values in the table by adding or subtracting the appropriate areas from 14, as we move toward or away from the t-value $t = 3$. For example, between $t = 2$ and $t = 3$, $R(t)$ decreases by 5/4, so $R(2) = 14 + \frac{5}{4}$.

5. [7 points] A local beet company, Dope Beets Inc., is developing a new beet with an adjustable growth rate for its many different customers. The growth rate of their new beet, measured in pounds per day, t days after a beet is planted, is given by

$$r(t) = \frac{5t^2}{t^k + t + 1},$$

for some adjustable value $k > 1$.

a. [4 points] Suppose a new beet initially weighs 2 pounds. Write an expression involving an integral for the weight, in pounds, of the beet t days after it is planted.

Answer: $2 + \int_0^t \frac{5x^2}{x^k + x + 1}\,\mathrm{d}x$

b. [3 points] Dope Beets Inc. wants to adjust the value of k such that a planted beet will never have infinite weight, even if the beet is allowed to grow forever. Which values of k would keep the weight finite? Give your answer as a value, list of values, or interval, as appropriate. No justification is required.

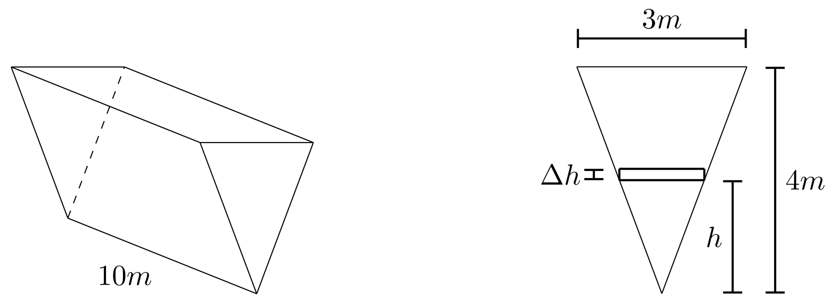
Solution: (Not required). Consider the integral

$$\int_1^\infty \frac{1}{x^{k-2}}\,\mathrm{d}x.$$

Let $p = k - 2$; this integral converges by the p-test when $p > 1$, or when $k > 3$. By (properly) using the direct or limit comparison tests, this shows the $\int_0^\infty \frac{5x^2}{x^k + x + 1}\,\mathrm{d}x$ (the total change in a beet's weight over all time) converges to a finite value.

Answer: $k > 3$

4. [15 points] A gas station needs to pump gas out of a subterranean tank. The tank is 10 meters in length, and has cross-sections shaped like isosceles triangles, with base 3 meters and height 4 meters. **The top of the tank is 15 meters below the surface of the earth**. Recall that $g = 9.8m/s^2$ is the gravitational constant.



Underground Tank

a. [5 points] Write an expression for the volume (in cubic meters) of a horizontal rectangular slice of gasoline at height h above the bottom of the tank, with thickness Δh. Your answer should not involve an integral.

Solution: The slice has volume $\ell w \Delta h$. The length is constant at $10m$, so we just need to find the width as a function of height. Using similar triangles, $\frac{w}{h} = \frac{3}{4}$ so $w = \frac{3}{4}h$. Therefore the slice volume is $\frac{15}{2}h\Delta h\ m^3$.

b. [3 points] Gasoline has a density of 800 kg/m^3. Write an expression for the weight (in newtons) of the slice of gasoline mentioned in part (a). Your answer should not involve an integral.

Solution: Weight is the force exerted on a mass due to gravity, and so weight is mg. We compute the mass using the density from the problem statement and the volume from a). This means $m = (800)(\frac{15}{2}h)\Delta h kg$. Then the weight is

$$(9.8)(800)\left(\frac{15}{2}h\right)\Delta h\ N$$

.

c. [4 points] Write an expression for the work (in joules) needed to pump the slice of gasoline mentioned above to the surface of the earth. Your answer should not involve an integral.

Solution: Work is force times distance traveled. If the slice is h meters from the bottom of the tank, then the slice travels $(4-h)$ meters to get to the top of the tank. Then it travels the 15 meters to get from the top of the tank to the ground. Therefore, the total distance traveled is $19-h$ meters. The force is the weight from b), so the work for a slice is:

$$W_{\text{slice}} = (19-h)(9.8)(800)\left(\frac{15}{2}h\right)\Delta h$$

d. [3 points] Write an integral for the total work (in joules) needed pump all of the gasoline to the surface of the earth.

Solution: We integrate our work slices to find the total work. Since the slices range from $h=0$ to $h=4$, these are the bounds, so the work is given by the integral

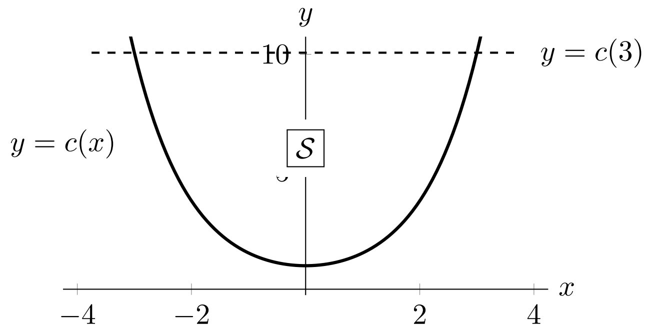
$$\int_0^4 (19-h)(9.8)(800)\left(\frac{15}{2}h\right)\, dh$$

2. [13 points]

Consider the function c defined for all real numbers x by the formula

$$c(x) = \frac{e^x + e^{-x}}{2}.$$

A portion of the graph of this "catenary" function is shown as the solid curve in the graph on the right.



Let $\mathcal{S}$ be the region bounded by the graph of $y = c(x)$ and the line $y = c(3)$. This region $\mathcal{S}$ is shown in the figure above.

a. [2 points] Write, but do **not** evaluate, an expression involving one or more integrals that gives the area of $\mathcal{S}$.

Solution: Note that $c(3) = \frac{e^3+e^{-3}}{2}$ is a constant and that the graph of $y = c(3)$ intersects the graph of $y = c(x)$ at $x = \pm 3$. The area of $\mathcal{S}$ is the area between the graphs above.

$$\text{Area} = \int_{-3}^{3} \frac{e^3 + e^{-3}}{2}\,dx - \int_{-3}^{3} \frac{e^x + e^{-x}}{2}\,dx = \int_{-3}^{3} \left(\frac{e^3 + e^{-3}}{2} - \frac{e^x + e^{-x}}{2} \right) dx$$

b. [5 points] A solid is obtained by rotating the region $\mathcal{S}$ about the x-axis.
Write, but do **not** evaluate, an expression involving one or more integrals that gives the volume of this solid.

Solution: Taking slices perpendicular to the x-axis, each slice is "washer"-shaped, and the volume of such a slice at x of thickness Δx is approximately $\pi\left((c(3))^2 - (c(x))^2\right)\Delta x$. The volume of the entire solid is then

$$\int_{-3}^{3} \pi\left((c(3))^2 - (c(x))^2\right) dx = \int_{-3}^{3} \pi \left[\left(\frac{e^3 + e^{-3}}{2} \right)^2 - \left(\frac{e^x + e^{-x}}{2} \right)^2 \right] dx$$

c. [3 points] Write, but do **not** evaluate, an expression involving one or more integrals that gives the arc length of the graph of $y = c(x)$ over the interval $-3 \le x \le 3$.
(Your answer should **not** involve any function names.)

Solution: We first compute c' to find $c'(x) = \dfrac{e^x - e^{-x}}{2}$.

Substituting this into the formula for the arc length of the graph of a function gives

$$\text{Arc length} = \int_{-3}^{3} \sqrt{1 + (c'(x))^2}\,dx = \int_{-3}^{3} \sqrt{1 + \left(\frac{e^x - e^{-x}}{2} \right)^2}\,dx.$$

d. [3 points] Will the midpoint rule with 2000 subdivisions give an underestimate or an overestimate of the value of $\displaystyle\int_{-3}^{0} c(x)\,dx$?

Circle your answer below. Then briefly explain your reasoning in the space on the right.

Circle one:

$\boxed{\textbf{Underestimate}}$

Overestimate

Neither (They are equal)

Cannot be determined

Briefly explain your reasoning.

Solution: Note that the second derivative of c is given by $c''(x) = \frac{e^x+e^{-x}}{2}$, which is positive for all values of x (since both e^x and e^{-x} are always positive). Since c'' is always positive, the function c is always concave up. Therefore every approximation of this definite integral using the midpoint rule (including MID(2000)) will be an underestimate.†

† Note that $c'' = c$. This is a very special property of this function c.

3. [12 points] Indicate whether you think each of the following series converges, diverges, or whether there is not enough information to determine convergence. You do not need to show your work for this page.

a. [3 points] Suppose $\sum_{n=1}^{\infty}(a_n + b_n)$ converges. Determine the convergence of $\sum_{n=1}^{\infty} a_n$.

CONVERGES DIVERGES $\boxed{\text{CANNOT TELL}}$

b. [3 points] Suppose $\sum_{n=1}^{\infty} a_n$ converges. Determine the convergence of $\sum_{n=1}^{\infty}(a_n + 4)$.

CONVERGES $\boxed{\text{DIVERGES}}$ CANNOT TELL

c. [3 points] Suppose $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$. Determine the convergence of $\sum_{n=1}^{\infty} \frac{a_n}{n}$.

$\boxed{\text{CONVERGES}}$ DIVERGES CANNOT TELL

d. [3 points] Determine the convergence of $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$.

$\boxed{\text{CONVERGES}}$ DIVERGES CANNOT TELL