Math 116 — Practice for Exam 1

Generated February 8, 2026

UMID: ____ Initials: ______

Instructor: ______________________________ Section Number: ____________

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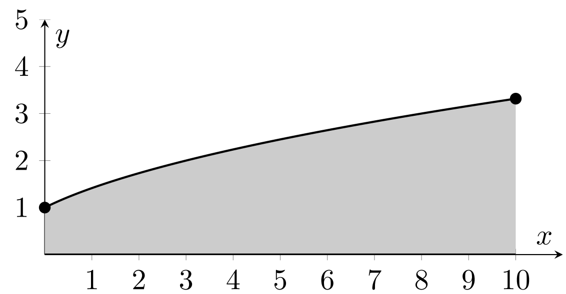
1. This exam has 10 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You are allowed notes written on two sides of a $3'' \times 5''$ note card. You are NOT allowed other resources, including, but not limited to, notes, calculators or other electronic devices.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. Problems may ask for answers in exact form. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2022	1	6	submarine	10	
Winter 2021	1	4	juice	10	
Fall 2023	1	1		15	
Fall 2025	1	2		14	
Fall 2018	1	3	tent	8	
Winter 2025	1	7		7	
Winter 2024	1	8		6	
Winter 2019	1	5		9	
Winter 2023	1	6	doughnut	16	
Fall 2017	1	10		12	
Total				107	

Recommended time (based on points): 96 minutes

6. [10 points] Denise and Trystan are undersea research scientists, and they are preparing to descend into the ocean in a newly-constructed submarine. The submarine's shape is given by rotating the region below the curve $y = \sqrt{x+1}$, above the x-axis, and between $x = 0$ and $x = 10$ (see figure) about the x-axis. Here, x and y are measured in meters.

Graph of $y = \sqrt{x+1}$ from $x = 0$ to $x = 10$



The density of the submarine is not constant, due to the advanced materials used in its construction. Instead, the density $p(x)$ varies, and is given by $p(x) = (x-5)^2 + 1$ kg/m^3.

a. [5 points] Write an expression for the **volume** of a slice of the submarine at position x and of thickness Δx. Include units.

b. [2 points] Write an expression for the **mass** of the slice you found in part (a). Include units.

c. [3 points] Write, but do not evaluate, an integral which gives the **total mass** of the submarine. Include units.

4. [10 points] After walking in the woods, Flora is making juice with the fruit she picked up at the next hour. The volume of juice in the jar (in gallons) t minutes after she starts making juice is given by the function

$$F(t) = \int_{\sin t}^{2t} \frac{50}{100 - \ln(x+2)}\, dx.$$

a. [3 points] Calculate $F'(t)$.

b. [3 points] What is the volume of juice (in gallons) in the jar when Flora starts making the juice? Briefly explain your answer using the function $F(t)$.

c. [4 points] Nile wants to know the volume of juice in the jar, yet she is confused by the function $F(t)$. She knows she can write $F(t)$ using $F'(t)$ and the initial volume of juice in the jar. Help her by rewriting $F(t)$ in the form

$$F(t) = \int_a^t \underline{\qquad\quad}\; d\underline{\qquad\quad} + \underline{\qquad\quad}.$$

Write the above integral with the blanks filled in, and also give the value of a.

1. [15 points] Let $f(x)$ be a differentiable function whose derivative $f'(x)$ is also differentiable and is always positive. Some values of $f(x)$ and $f'(x)$ are given in the table below:

x	1	2	3	4	5	6
$f(x)$	2	7	8	12	15	17
$f'(x)$	3	2	6	11	4	5

Additionally, you are given that $\displaystyle\int_2^4 \frac{f(x)}{x}\,dx = 9.$

Compute the exact value of the following integrals. If there is not enough information provided to determine the value of the integral, write "NEI" and clearly indicate why. Show all of your work.

a. [5 points] $\displaystyle\int_1^2 \left(f'(t)+4\right) e^{f(t)+4t}\,dt$

Answer: ______________________

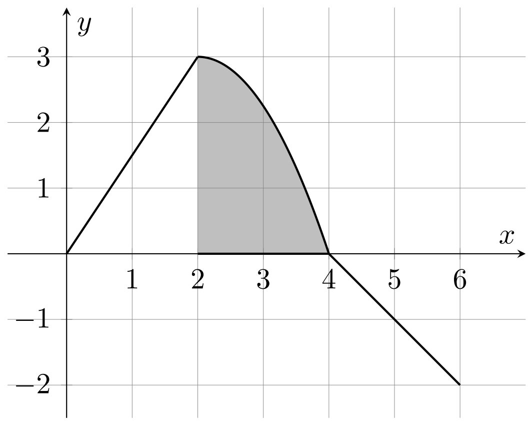
b. [5 points] $\displaystyle\int_2^4 f'(x)\ln x\,dx$

Answer: ______________________

c. [5 points] $\displaystyle\int_{\ln 2}^{\ln 4} f(e^x)\,dx$

Answer: ______________________

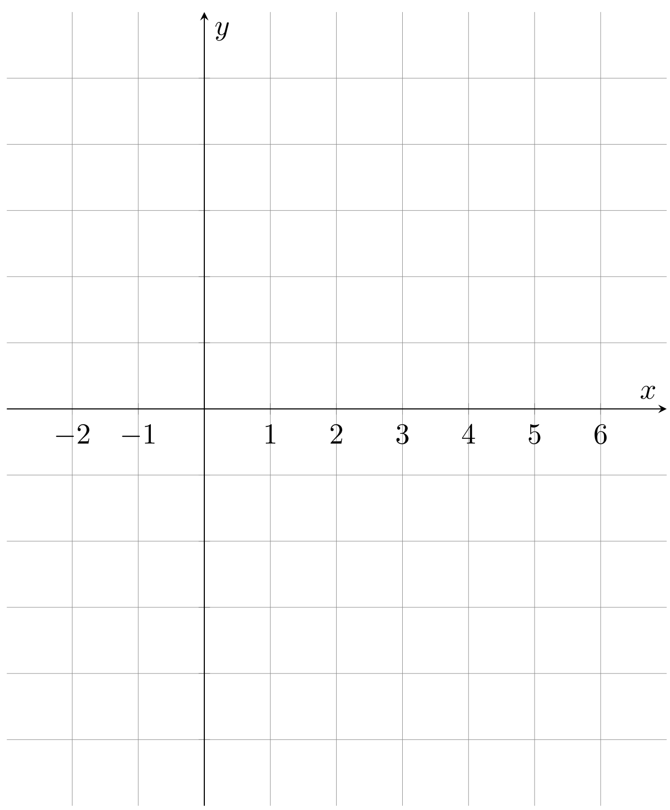
2. [14 points] A function $g(x)$, which is defined for all real numbers, is graphed on the interval $[0, 6]$ below. Note that $g(x)$ is linear on the intervals $(0, 2)$ and $(4, 6)$, and that the shaded region has area 4. Additionally, $g(x)$ is an **<u>odd function</u>**.



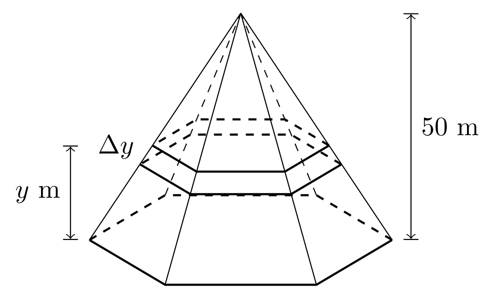
a. [4 points] The function $g(x)$ has a continuous antiderivative, $G(x)$, which satisfies $G(2) = 1$. Complete the following table of values for $G(x)$.

x	-2	0	2	4	6
$G(x)$			1		

b. [10 points] Sketch a graph of $G(x)$ on the interval $[-2, 6]$ using the axes provided. Make sure to <u>clearly label the scale on the y-axis</u> and also make it clear where $G(x)$ is increasing or decreasing, and where $G(x)$ is concave up, concave down, or linear.



3. [8 points]
Consider a tent that is 50 meters tall whose base is a regular hexagon (i.e. a 6-sided polygon with equal length sides and equal angles) and whose horizontal cross-sections are also regular hexagons.
(See figure on the right.)
Suppose the perimeter of the base is 72 meters.
Let $P(y)$ be the perimeter, in meters, of a horizontal cross section y meters above the ground.



a. [2 points] It turns out that $P(y)$ is a linear function of the variable y. (You do not need to verify this.) Find a formula for $P(y)$.

Answer: $P(y) =$ ______________________

b. [3 points] The area of a regular hexagon with perimeter p is equal to $\frac{\sqrt{3}}{24}p^2$.

Write an expression that gives the approximate volume, in cubic meters, of a horizontal slice of the region inside the tent that is Δy meters thick and y meters above the ground. (Assume here that Δy is small but positive.) Your expression should not involve any integrals.

Answer: Volume of slice $\approx$ ______________________

c. [3 points] Write, but do not evaluate, an expression involving one or more integrals that gives the total volume, in cubic meters, inside the tent.

Answer: Volume = ______________________

7. [7 points] Use the partial fraction decomposition

$$\frac{x^2+11x-6}{(2-x)(x^2+1)} = \frac{4}{2-x} + \frac{3x-5}{x^2+1}$$

to evaluate the following indefinite integral, showing all of your work.

$$\int \frac{x^2+11x-6}{(2-x)(x^2+1)}\, \mathrm{d}x$$

Answer: ______________________________

8. [6 points] Each part below describes a twice differentiable function and one or more approximations of its integral. For each of the following statements, determine if the statement is **ALWAYS** true, **SOMETIMES** true, or **NEVER** true, and circle the appropriate answer.
No justification is required.

a. [1 point] If $A'(x) > 0$ for all x, then LEFT(4) $\leq \int_{-1}^{1} A(x)\,dx$.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

b. [1 point] If $B'(x) > 0$ for all x, then TRAP(4) $\leq \int_{-1}^{1} B(x)\,dx$.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

c. [1 point] If $C''(x) > 0$ for all x, then TRAP(4) $\leq \int_{-1}^{1} C(x)\,dx$.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

d. [1 point] If $D(x)$ is odd and MID(4) approximates $\int_{-1}^{1} D(x)\,dx$, then MID(4) $= 0$.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

e. [1 point] If $E'(x) > 0$ and $E''(x) < 0$ for all x, then $\int_{-1}^{1} E(x)\,dx \leq$ MID(2) $\leq$ RIGHT(2).

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

f. [1 point] If $F(x)$ is not constant, then RIGHT(3) approximates the integral $\int_{-1}^{1} F(x)\,dx$ more accurately than RIGHT(2).

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

5. [9 points] Before calculators existed, it was a difficult task for scientists and engineers to compute values of special functions like logarithm. In this problem, we will use simple arithmetic operations to approximate the value of $\ln 2$.
Note that for $x > 0$,

$$\ln x = \int_1^x \frac{1}{t}\, dt.$$

a. [3 points] Approximate the integral $\displaystyle\int_1^2 \frac{1}{t}\, dt$ using LEFT(4). Write out each term in your sum.

Answer: ____________________

b. [3 points] Which of the following are equal to the LEFT(n) approximations for $\displaystyle\int_1^2 \frac{1}{t}\, dt$? Circle the one best answer.

i. $\displaystyle \frac{1}{n}\sum_{i=0}^{n-1} \frac{1}{i}$

iv. $\displaystyle \frac{1}{n}\sum_{i=1}^{n} \frac{1}{i}$

ii. $\displaystyle \frac{1}{n}\sum_{i=0}^{n} \frac{1}{1+i/n}$

v. $\displaystyle \frac{1}{n}\sum_{i=n}^{n} \frac{1}{1+i/n}$

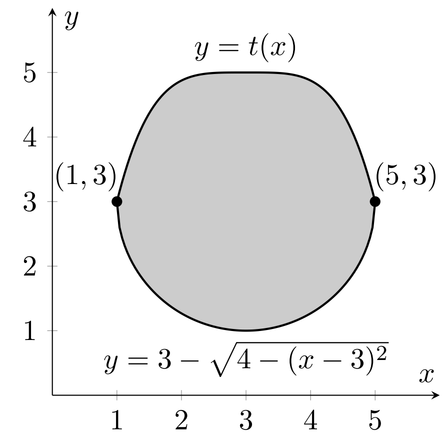
iii. $\displaystyle \frac{1}{n}\sum_{i=0}^{n-1} \frac{1}{1+i/n}$

vi. $\displaystyle \frac{1}{n}\sum_{i=0}^{n} \frac{1}{1+1/n}$

c. [3 points] How many subintervals would be needed so that a scientist who lived before calculators were invented would be certain that the resulting left-hand Riemann sum approximates $\ln(2)$ to within 0.01? Justify your answer.

Answer: ____________________

6. [16 points] Ariana, a baker at the vegan bakery VCorp, is designing a new doughnut. The cross section of the doughnut is shown below, where the units of both x and y are cm.



The top of the cross section is given by the function $y = t(x)$ and the bottom is given by the semicircle $y = 3 - \sqrt{4 - (x-3)^2}$. Ariana is experimenting with two ideas for the doughnut.

a. [6 points] Her first idea is to rotate the cross section around the y-axis. Write an integral that gives the volume of the resulting doughnut. Do not evaluate your integral. Your answer may involve the function t, but it should not involve t^{-1} (the inverse of t).

Answer: ______________________

b. [6 points] Her second idea is to rotate the cross section around the x-axis. Write an integral that gives the volume of the resulting doughnut. Do not evaluate your integral. Your answer may involve the function t, but it should not involve t^{-1} (the inverse of t).

Answer: ______________________

c. [4 points] As is tradition at VCorp, they are planning to wrap a ribbon around the cross section of the doughnut. Write an expression involving one or more integrals that gives the total perimeter of the cross section. Do not evaluate the integrals in your expression.

Answer: ______________________

10. [12 points] For each of the questions below, circle all of the available correct answers.
Circle "NONE OF THESE" if none of the available choices are correct.
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

a. [4 points] Suppose a function f and both its derivative f' and second derivative f'' are defined and continuous on the entire real line $(-\infty, \infty)$. Which of the following functions must be antiderivatives of the function $t^2 f'(t)$ on $(-\infty, \infty)$?

i. $\int_1^t 2y f''(y)\, dy$ ii. $5 + \int_{-3}^t w^2 f'(w)\, dw$ iii. $0.25 \int_0^{2t} x^2 f'(0.5x)\, dx$

iv. $t^2 f(t) + \int_t^2 2x f(x)\, dx$ v. $f'(1) + \int_1^4 t^2 f'(t)\, dt$ vi. NONE OF THESE

b. [4 points] Suppose that g is a function that is continuous, negative, and decreasing on the interval $[-4, 4]$ and that n is a positive integer.
Consider the definite integral $\int_{-4}^4 g(x)\, dx$ and the four approximations of $\int_{-4}^4 g(x)\, dx$ given by RIGHT(n), LEFT(n), TRAP(n), MID(n).
Which of the following could be true about the relationships between these five numbers?

i. $\text{TRAP}(n) < \int_{-4}^4 g(x)\, dx$ ii. $\text{TRAP}(n) > \int_{-4}^4 g(x)\, dx$

iii. $\text{MID}(n) < \int_{-4}^4 g(x)\, dx$ iv. $\text{MID}(n) > \int_{-4}^4 g(x)\, dx$

v. $\text{RIGHT}(n) < \int_{-4}^4 g(x)\, dx < \text{LEFT}(n)$ vi. $\text{TRAP}(n) = \text{MID}(n)$

vii. $\text{LEFT}(n) < \int_{-4}^4 g(x)\, dx < \text{RIGHT}(n)$ viii. NONE OF THESE

c. [4 points] Suppose Q is a continuous function. A circular metal plate in the xy-plane with radius 10 cm has density $Q(r)$ grams per square centimeter at a distance of r centimeters from the center of the plate. Which of the following statements must be true about this plate?

i. The total mass of the plate is $100\pi \cdot Q(10)$ grams.

ii. The total mass of the plate is $\int_{-10}^{10} 2\pi r \cdot Q(r)\, dr$ grams.

iii. The mass, in grams, of a very thin horizontal slice of the plate of height Δy cm located y cm above the center of the plate is approximately $Q(y)$ times the area, in cm^2, of the slice.

iv. The total mass of the plate is $\int_0^{10} \pi r^2 \cdot Q(r)\, dr$ grams.

v. NONE OF THESE