Math 116 — Practice for Exam 2

Generated March 21, 2026

UMID: ______________________________ Initials: ______

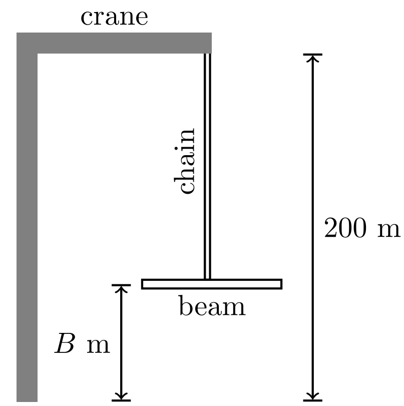
Instructor: ______________________ Section Number: ________

1. This exam has 11 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You are allowed notes written on two sides of a $3'' \times 5''$ note card. You are NOT allowed other resources, including, but not limited to, notes, calculators or other electronic devices.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. Problems may ask for answers in exact form. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2017	1	9	chain	8	
Fall 2021	2	9	gardening	20	
Winter 2025	2	2		4	
Fall 2020	2	7	cat trumpet	16	
Fall 2025	1	9		12	
Fall 2024	2	10		12	
Winter 2003	3	9	nautilus	6	
Fall 2020	2	4	redwoods	13	
Winter 2017	3	2	disc tower	7	
Fall 2010	3	7		14	
Winter 2025	2	5		6	
Total				118	

Recommended time (based on points): 114 minutes

9. [8 points]
During the construction of a skyscraper, a 200 meter tall crane lifts a steel beam from the ground to a height of 175 meters. The steel beam has a mass of 50 kilograms. The crane has a chain that is also made of steel, and the chain has a mass of 15 kilograms per meter. The total length of the chain is 200 meters, but as the beam is lifted, the crane no longer needs to lift any of the chain that has already been "reeled in", i.e. has already reached the top of the crane.



For this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s^2 .

a. Write an expression in terms of B that gives the total mass, in kilograms, of the steel beam together with the chain that has not yet been reeled in at the moment that the steel beam is B meters above the ground.

b. Assuming ΔB is very small but positive, write an expression in terms of B that approximates the work done by the crane in lifting the steel beam up ΔB meters starting from a height of B meters above the ground. Assume that the weight of the chain being lifted is constant over this very short distance. Include units.

c. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total work that must be done by the crane in order to lift the steel beam from the ground to a height of 175 meters. Include units.

9. [20 points] Otto would like to landscape his yard, so he contacts the company Granville's Calculate-Yourself Gardening. Granville's company provides each potential customer with a list of possible equations and improper integrals guiding the landscaping of the yard. Granville's company also offers a discount to customers that can correctly solve the integrals. Otto, strapped for cash, desperately wants to solve the equations Granville has sent him.

a. [3 points] Granville has also informed Otto that he can plant a maple tree in the back of the yard that is initially 2 meters tall. Granville estimates that the maple tree will grow at an instantaneous rate of

$$M(t) = \frac{12t}{e^t}$$

meters per year t years after it is planted. Write an integral that gives the height of the tree t years after it is planted. Your answer should not involve the letter M.

b. [7 points] Determine the maximum height that the maple tree will grow to.

9. (continued)

c. [10 points] Granville tells Otto that he can get a truck to come and dispense dirt into the yard for 2 hours. The instantaneous rate that the truck will dispense dirt t hours after the truck arrives is

$$D(t) = \frac{(\sin(t))^2}{t^{5/2}\sqrt{2-t}}$$

pounds per minute. Show that $\displaystyle\int_0^2 D(t)dt$ converges. Justify all of your work.

Hint 1: Try splitting this into 2 integrals, one from 0 to 1, and the other from 1 to 2.

Hint 2: You may want to use the fact that $\sin t \le t$ *for* $t \ge 0$.

2. [4 points] Compute the following limit. Fully justify your answer including using **proper limit notation**.

$$\lim_{x\to 1} \frac{\sin(\ln(x))}{x^2-1}$$

Answer: $\lim_{x\to 1} \frac{\sin(\ln(x))}{x^2-1} =$ ______________________________

3. [7 points] **Compute** the value of the following improper integral if it converges.
If it does not converge, use a **direct computation** of the integral to show its divergence.
Be sure to show your full computation, and be sure to use **proper notation**.

$$\int_0^\infty \frac{x}{e^{3x^2}}\,\mathrm{d}x$$

Circle one: **Diverges** **Converges to** ________________

7. [16 points] Gabriel the aspiring jazz musician owns a number of cats and kittens which he lets wander his neighborhood. When he wants to feed them, he blows his trusty cat trumpet, and waits for them to come running.

a. [6 points] The probability density function for the time t, in minutes, that it takes for Miles the cat to arrive is given by $m(t)$ where

$$m(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{5} & \text{for } 0 \leq t \leq a \\ \frac{1}{5}e^{-t+a} & \text{for } t > a \end{cases}$$

for some constant a. Find the value of a.

Answer: ________________________

b. [7 points] Find the mean time in minutes that it takes Miles to arrive. You should evaluate any integrals or limits in your expression. You may give your answer in terms of a, but not in terms of m. You are not required to simplify your answer.

Answer: ________________________

c. [3 points] The cumulative distribution function for the amount of time that it takes for Ella the kitten to arrive is given by $E(t)$. Gabriel knows that 18% of the time Ella arrives in less than 2 minutes, and that 40% of the time she takes more than 6 minutes to arrive. Use this information to find the value of $E(6) - E(2)$.

Answer: ________________________

9. [12 points]

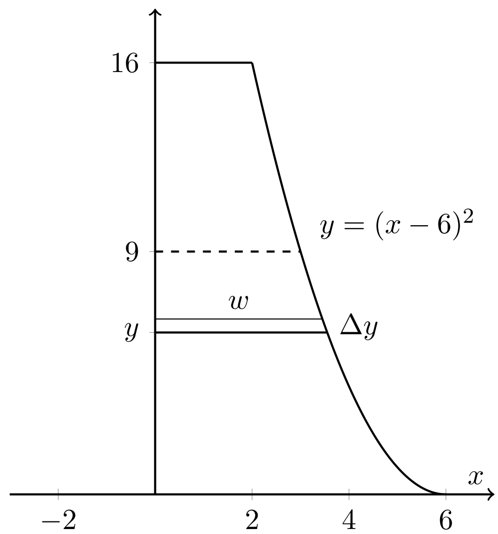
A big tank at a chemical factory is formed by rotating the region in the first quadrant bounded by $y = 16$, and

$$y = (x - 6)^2,$$

around the y-axis. All distances are measured in meters. The tank is filled with liquid chemicals up to $y = 9$ meters, as shown by the dashed line in the plot to the right. Due to sedimentation, the liquid has a varying density of

$$f(y) = 3 - 0.1y \quad \text{kg/m}^3$$

at height y. Workers at the factory will pump the chemicals out through the top of the tank. You may assume that the acceleration due to gravity is $g = 9.8\text{m/s}^2$.



a. [2 points] Consider the thin horizontal strip of the region depicted above, which is located y meters above the x-axis. It has horizontal length w and a small thickness Δy. Find a formula for w in terms of y.

Answer: $w =$ ________________

b. [4 points] When the strip above is rotated around the y-axis, it forms a thin **disk**. Write an expression which approximates the **mass** of that disk. Your answer should not involve any integrals, and you should express your answer in terms of y, and Δy. **Include units.**

Answer: ________________ **Units:** ________

c. [3 points] Write an expression which approximates the work needed to lift the thin disk described in part **b** to the top of the tank. Your answer should not involve any integrals, and you should express your answer in terms of y, and Δy. **Include units.**

Answer: ________________ **Units:** ________

d. [3 points] Write an expression involving one or more integrals representing the work needed to pump all the liquid chemicals to top of the tank, using the same units as in part **c**. **Do not** evaluate any integrals in your expression.

Answer: ________________

10. [12 points]

a. [6 points] For each of the following sequences or series below, determine whether they must converge, must diverge, or whether there is not enough information. Circle your answers. No justification is required.

(i) $a_n = \int_1^n f(x)\,dx$ where $f(x) \geq 0$, $f'(x) \leq 0$, and $\lim_{x\to\infty} f(x) = 0$.

Circle one: **Converges** **Diverges** **Not Enough Information**

(ii) $\sum_{n=1}^{\infty} (-1)^n (1 + s^{-n})$ where s is a positive real number.

Circle one: **Converges** **Diverges** **Not Enough Information**

(iii) $\sum_{n=1}^{\infty} \frac{\sin n}{k^n}$ where k is a real number with $k > e$.

Circle one: **Converges** **Diverges** **Not Enough Information**

b. [6 points] For each of the following sequences, defined for $n \geq 1$, decide whether the sequence is monotone increasing, monotone decreasing, or not monotone, and whether it is bounded or unbounded. Circle your answers. No justification is required.

(i) $b_n = \frac{(-1)^n}{2n}$

*Circle **all** which apply:*

Monotone Increasing **Monotone Decreasing** **Not Monotone**

Bounded **Unbounded**

(ii) $c_n = e^n \cos\left(\frac{1}{n}\right)$

*Circle **all** which apply:*

Monotone Increasing **Monotone Decreasing** **Not Monotone**

Bounded **Unbounded**

(iii) $d_n = \int_2^{2n} \frac{1}{(x-1)^2}\,dx$

*Circle **all** which apply:*

Monotone Increasing **Monotone Decreasing** **Not Monotone**

Bounded **Unbounded**

9. (6 pts) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. How much volume is enclosed by all the chambers? Assume for simplicity that there are infinitely many chambers. Show your work.

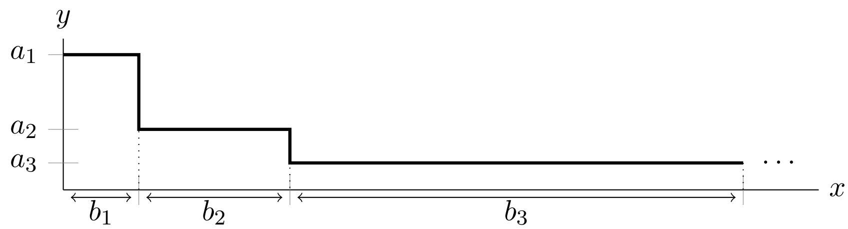
4. [13 points] Rafael finds details of another of TimberCorp's logging operations, this time in a forest of redwoods which initially has 50,000 trees. TimberCorp plans to, at the start of each year, cut down 10% of the trees in the forest, and then over the course of the year replant k trees.

a. [5 points] Let R_n be the number of trees in the forest **at the end** of the nth year of the logging operation. Find expressions for R_1 and R_2. Your answers may involve k. **You do not need to simplify your answers**.

b. [5 points] Find a **closed form** expression for R_n. Closed form means your answer should not include ellipses or sigma notation, and should NOT be recursive. **You do not need to simplify your closed form answer.**

c. [3 points] Rafael wants the number of trees in the forest at the end of a year to tend towards 70,000 in the long run (i.e. after many many years). What value should he choose for k to ensure this happens?

2. [7 points] The region depicted below consists of infinitely many adjacent rectangles. (Only the first three rectangles are actually shown, and they are not necessarily drawn to scale.) For $n = 1, 2, 3, \ldots$, the nth rectangle has height $a_n = \dfrac{1}{5^{n/2}}$ and width $b_n = n!$.



a. [5 points] Write an infinite series that gives the total volume of the solid formed by rotating the entire region (all of the rectangles) around the x-axis.

b. [2 points] Does the infinite series that gives the total volume of the solid formed by rotating the entire region (all of the rectangles) around the x-axis converge or diverge?

CIRCLE ONE: **Converges** **Diverges**

State the name of the test you would use to justify your answer. If you would use the comparison test or limit comparison also give a valid comparison series. You do not need to actually write out a full justification. (If you do not know the name of the test you would use, state the test itself.)

7. [14 points] For each of the following sequences

1.Compute $\lim_{n\to\infty} a_n$.

2.Decide if $\sum_{n=0}^{\infty} a_n$ converges or diverges. Circle your answer.

Support your answer by stating the test(s) or facts you used to prove convergence or divergence , and show complete work and justification.

a. [4 points]

$$a_n = \left(\frac{-1}{\pi}\right)^n \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \qquad \text{Diverges}$$

b. [4 points]

$$a_n = \frac{n^2+2}{1+4n^2} \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \qquad \text{Diverges}$$

c. [6 points]

$$a_n = \frac{n}{\sqrt{n^4+5}} \qquad \lim_{n\to\infty} a_n = \_\_\_\_\_\_ \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \qquad \text{Diverges}$$

5. [6 points] Determine if the following series converges or diverges, and circle the corresponding word. **Fully justify** your answer including using **proper notation** and **showing mechanics** of any tests you use.

$$\sum_{n=1}^{\infty} \frac{4^n}{n!}$$

Circle one: **Converges** **Diverges**

Justification: