MATH 116 — PRACTICE FOR EXAM 2

Generated March 21, 2026

UMID: **SOLUTIONS** ______ INITIALS: ______

INSTRUCTOR: ______ SECTION NUMBER: ______

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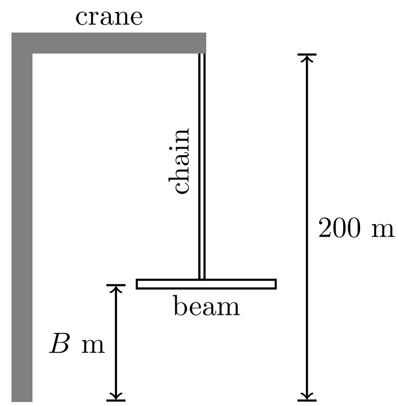
1. This exam has 11 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You are allowed notes written on two sides of a $3'' \times 5''$ note card. You are NOT allowed other resources, including, but not limited to, notes, calculators or other electronic devices.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. Problems may ask for answers in exact form. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2017	1	9	chain	8	
Fall 2021	2	9	gardening	20	
Winter 2025	2	2		4	
Fall 2020	2	7	cat trumpet	16	
Fall 2025	1	9		12	
Fall 2024	2	10		12	
Winter 2003	3	9	nautilus	6	
Fall 2020	2	4	redwoods	13	
Winter 2017	3	2	disc tower	7	
Fall 2010	3	7		14	
Winter 2025	2	5		6	
Total				118	

Recommended time (based on points): 114 minutes

9. [8 points]
During the construction of a skyscraper, a 200 meter tall crane lifts a steel beam from the ground to a height of 175 meters. The steel beam has a mass of 50 kilograms. The crane has a chain that is also made of steel, and the chain has a mass of 15 kilograms per meter. The total length of the chain is 200 meters, but as the beam is lifted, the crane no longer needs to lift any of the chain that has already been "reeled in", i.e. has already reached the top of the crane.



For this problem, you may assume the acceleration due to gravity is $g = 9.8\ \text{m/s}^2$.

a. Write an expression in terms of B that gives the total mass, in kilograms, of the steel beam together with the chain that has not yet been reeled in at the moment that the steel beam is B meters above the ground.

Solution: If the steel beam is B meters above the ground, that means that a length of B meters of chain has been reeled in, so $(200-B)$ meters of chain remains. The mass of this remaining chain is (15 kilograms per meter) $\cdot$ ($200 - B$ meters) $= 3000 - 15B$ kilograms. We add to this the mass of the steel beam to find a total mass of

$$\text{Mass} = 50 + 3000 - 15B \text{ kilograms.}$$

b. Assuming ΔB is very small but positive, write an expression in terms of B that approximates the work done by the crane in lifting the steel beam up ΔB meters starting from a height of B meters above the ground. Assume that the weight of the chain being lifted is constant over this very short distance. Include units.

Solution: The force due to gravity is the weight. At the moment the steel beam is B meters above the ground, the weight of the beam together with the chain that has not yet been reeled in is

$$\text{Weight} = (\text{mass})(g) = (3050 - 15B \text{ kilograms})(9.8 \text{ m/s}^2) = 29890 - 147B \text{ Newtons.}$$

The work to lift the steel beam over the small distance ΔB is then approximately

$$(\text{Force}) \cdot \Delta B = (29890 - 147B)\Delta B \text{ Joules.}$$

c. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total work that must be done by the crane in order to lift the steel beam from the ground to a height of 175 meters. Include units.

Solution: Using our answer from part **b.** above, summing over the entire path of the beam, and taking the limit as ΔB approaches 0, we find that the total work is

$$\text{Work} = \int_0^{175} g(3050 - 15B)\, dB = \int_0^{175} (29890 - 147B)\, dB \text{ Joules.}$$

9. [20 points] Otto would like to landscape his yard, so he contacts the company Granville's Calculate-Yourself Gardening. Granville's company provides each potential customer with a list of possible equations and improper integrals guiding the landscaping of the yard. Granville's company also offers a discount to customers that can correctly solve the integrals. Otto, strapped for cash, desperately wants to solve the equations Granville has sent him.

a. [3 points] Granville has also informed Otto that he can plant a maple tree in the back of the yard that is initially 2 meters tall. Granville estimates that the maple tree will grow at an instantaneous rate of

$$M(t) = \frac{12t}{e^t}$$

meters per year t years after it is planted. Write an integral that gives the height of the tree t years after it is planted. Your answer should not involve the letter M.

Solution:

$$2 + \int_0^t \frac{12s}{e^s} ds.$$

b. [7 points] Determine the maximum height that the maple tree will grow to.

Solution: Let us first calculate the integral in (a) using integration by parts ($u = 12s, dv = e^{-s}ds$):

$$\begin{aligned}
\int_0^t \frac{12s}{e^s} ds &= -12se^{-s}|_0^t + 12 \int_0^t e^{-s} ds \\
&= -12te^{-t} + 12\Big[- e^{-s}|_0^t \Big] \\
&= -12te^{-t} - 12e^{-t} + 12
\end{aligned}$$

Since the tree is always growing, the maximum height is

$$\begin{aligned}
2 + \int_0^\infty \frac{12t}{e^t} dt &= 2 + \lim_{t\to\infty} \int_0^t \frac{12s}{e^s} ds \\
&= 2 + \lim_{t\to\infty} 12 - 12e^{-t} - 12te^{-t} \\
&= 14 - \lim_{t\to\infty} \frac{12t}{e^t} \\
&\overset{\text{L'H}\frac{\infty}{\infty}}{=} 14 - \lim_{t\to\infty} \frac{12}{e^t} \\
&= 14.
\end{aligned}$$

So, the maximum height of the tree is 12 meters.

9. (continued)

c. [10 points] Granville tells Otto that he can get a truck to come and dispense dirt into the yard for 2 hours. The instantaneous rate that the truck will dispense dirt t hours after the truck arrives is

$$D(t) = \frac{(\sin(t))^2}{t^{5/2}\sqrt{2-t}}$$

pounds per minute. Show that $\displaystyle\int_0^2 D(t)dt$ converges. Justify all of your work.

Hint 1: Try splitting this into 2 integrals, one from 0 to 1, and the other from 1 to 2.

Hint 2: You may want to use the fact that $\sin t \le t$ *for* $t \ge 0$.

Solution: Following the hint,

$$\int_0^2 D(t)dt = \int_0^1 D(t)dt + \int_1^2 D(t)dt.$$

Starting with the first integral, note that on $[0,1]$,

$$D(t) \le \frac{t^2}{t^{5/2}} \le t^{-1/2}$$

By the p-test ($p = \frac{1}{2}$), $\displaystyle\int_0^1 \frac{1}{\sqrt{t}}dt$ converges, so by the comparison test, $\displaystyle\int_0^1 D(t)dt$ converges.

Next, we perform a change of variables on the second integral($w = 2-t$):

$$\begin{aligned}
\int_1^2 D(t)dt &= \lim_{b\to 2}\int_1^b D(t)dt \\
&= \lim_{b\to 2} -\int_1^{2-b} \frac{(\sin(2-w))^2}{(2-w)^{5/2}\sqrt{w}}dw \\
&= \int_0^1 \frac{(\sin(2-w))^2}{(2-w)^{5/2}\sqrt{w}}dw
\end{aligned}$$

On the interval $[0,1]$,

$$\frac{(\sin(2-w))^2}{(2-w)^{5/2}\sqrt{w}} \le \frac{1}{\sqrt{w}}$$

By the p-test ($p = \frac{1}{2}$), $\displaystyle\int_0^1 \frac{1}{\sqrt{w}}dw$ converges. So, by the comparison test, $\displaystyle\int_1^2 D(t)dt$ converges. As both parts converge, we have verified that the integral $\displaystyle\int_0^2 D(t)dt$ converges.

2. [4 points] Compute the following limit. Fully justify your answer including using **proper limit notation**.

$$\lim_{x\to 1} \frac{\sin(\ln(x))}{x^2-1}$$

Solution: Since $\ln 1 = 0$, the limit is in the indeterminate form "$\frac{0}{0}$". We have

$$\lim_{x\to 1} \frac{\sin(\ln(x))}{x^2-1} \overset{\text{L'H}}{=} \lim_{x\to 1} \frac{\frac{\cos(\ln(x))}{x}}{2x} = \frac{\cos(\ln(1))}{2(1)^2} = \frac{\cos(0)}{2} = \frac{1}{2}.$$

Answer: $\lim_{x\to 1} \frac{\sin(\ln(x))}{x^2-1} =$ $\frac{1}{2}$

3. [7 points] **Compute** the value of the following improper integral if it converges.
If it does not converge, use a **direct computation** of the integral to show its divergence.
Be sure to show your full computation, and be sure to use **proper notation**.

$$\int_0^\infty \frac{x}{e^{3x^2}}\,\mathrm{d}x$$

Solution: By definition,

$$\int_0^\infty \frac{x}{e^{3x^2}}\,\mathrm{d}x = \lim_{b\to\infty} \int_0^b \frac{x}{e^{3x^2}}\,\mathrm{d}x.$$

Set $u = 3x^2$. Then $\mathrm{d}u = 6x\,\mathrm{d}x$. Using the method of substitution,

$$\int \frac{x}{e^{3x^2}}\,\mathrm{d}x = \frac{1}{6}\int e^{-u}\,\mathrm{d}u = -\frac{e^{-u}}{6} + C = -\frac{e^{-3x^2}}{6} + C.$$

Therefore,

$$\begin{aligned}
\int_0^\infty \frac{x}{e^{3x^2}}\,\mathrm{d}x &= \lim_{b\to\infty} \int_0^b \frac{x}{e^{3x^2}}\,\mathrm{d}x \\
&= \lim_{b\to\infty} \left[-\frac{e^{-3x^2}}{6}\Bigg|_0^b \right] \\
&= \lim_{b\to\infty} \left[-\frac{e^{-3b^2}}{6} + \frac{1}{6} \right] \\
&= 0 + \frac{1}{6} \\
&= \frac{1}{6}.
\end{aligned}$$

Circle one: **Diverges** $\boxed{\textbf{Converges to } \frac{1}{6}}$

7. [16 points] Gabriel the aspiring jazz musician owns a number of cats and kittens which he lets wander his neighborhood. When he wants to feed them, he blows his trusty cat trumpet, and waits for them to come running.

a. [6 points] The probability density function for the time t, in minutes, that it takes for Miles the cat to arrive is given by $m(t)$ where

$$m(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{5} & \text{for } 0 \leq t \leq a \\ \frac{1}{5}e^{-t+a} & \text{for } t > a \end{cases}$$

for some constant a. Find the value of a.

Solution: Since $m(t)$ is a pdf, we must have $\displaystyle\int_{-\infty}^{\infty} m(t)\, dt = 1$, and so

$$\begin{aligned} \frac{1}{5}a + \int_a^{\infty} \frac{1}{5}e^{-t+a}\, dt &= 1 \\ \frac{1}{5}a + \lim_{b\to\infty} \int_a^{b} \frac{1}{5}e^{-t+a}\, dt &= 1 \\ \frac{1}{5}a - \lim_{b\to\infty} \frac{1}{5}e^{-t+a}\Big|_a^b\, dt &= 1 \\ \tfrac{1}{5}a + \tfrac{1}{5} &= 1 \end{aligned}$$

and so $a = 5 - 1 = 4$.

b. [7 points] Find the mean time in minutes that it takes Miles to arrive. You should evaluate any integrals or limits in your expression. You may give your answer in terms of a, but not in terms of m. You are not required to simplify your answer.

Solution: The mean time is given by

$$\begin{aligned} \int_{-\infty}^{\infty} tm(t)\, dt &= \int_0^a \frac{t}{5}\, dt + \int_a^{\infty} \frac{t}{5}e^{-t+a}\, dt \\ &= \frac{a^2}{10} + \frac{1}{5}\lim_{b\to\infty} \int_a^b te^{-t+a}\, dt \\ &= \frac{a^2}{10} + \frac{1}{5}\lim_{b\to\infty} \left(-te^{-t+a}\Big|_a^b + \int_a^b e^{-t+a}\, dt \right) \\ &= \frac{a^2}{10} + \frac{1}{5}\lim_{b\to\infty} \left(\left(-te^{-t+a} - e^{-t+a}\right)\Big|_a^b \right) \\ &= \frac{a^2}{10} + \frac{1}{5}(a+1) \end{aligned}$$

c. [3 points] The cumulative distribution function for the amount of time that it takes for Ella the kitten to arrive is given by $E(t)$. Gabriel knows that 18% of the time Ella arrives in less than 2 minutes, and that 40% of the time she takes more than 6 minutes to arrive. Use this information to find the value of $E(6) - E(2)$.

Solution: From the given information, we see that $E(2) = 0.18$ and $E(6) = 1 - 0.4 = 0.6$. Therefore $E(6) - E(2) = 0.6 - 0.18 = 0.42$.

Answer: **0.42**

9. [12 points]

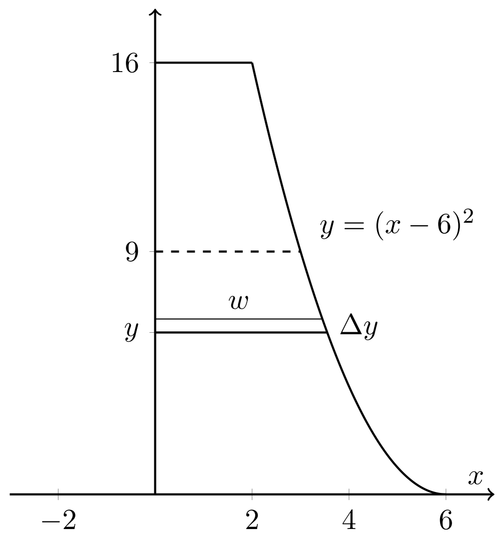
A big tank at a chemical factory is formed by rotating the region in the first quadrant bounded by $y = 16$, and

$$y = (x - 6)^2,$$

around the y-axis. All distances are measured in meters. The tank is filled with liquid chemicals up to $y = 9$ meters, as shown by the dashed line in the plot to the right. Due to sedimentation, the liquid has a varying density of

$$f(y) = 3 - 0.1y \quad \text{kg/m}^3$$

at height y. Workers at the factory will pump the chemicals out through the top of the tank. You may assume that the acceleration due to gravity is $g = 9.8\text{m/s}^2$.



a. [2 points] Consider the thin horizontal strip of the region depicted above, which is located y meters above the x-axis. It has horizontal length w and a small thickness Δy. Find a formula for w in terms of y.

Solution: We know that $y = (w - 6)^2$ and $w \leq 6$, so $w = 6 - \sqrt{y}$.

Answer: $w =$ $6 - \sqrt{y}$

b. [4 points] When the strip above is rotated around the y-axis, it forms a thin **disk**. Write an expression which approximates the **mass** of that disk. Your answer should not involve any integrals, and you should express your answer in terms of y, and Δy. **Include units.**

Solution: The volume of a slice is $\pi w^2 \Delta y$. To find the mass of a slice, we must multiply by the density, giving us $\pi w^2 f(y)\Delta y = \pi(6 - \sqrt{y})^2(3 - 0.1y)\Delta y$.

Answer: $\pi(6 - \sqrt{y})^2(3 - 0.1y)\Delta y$ **Units:** kg

c. [3 points] Write an expression which approximates the work needed to lift the thin disk described in part **b** to the top of the tank. Your answer should not involve any integrals, and you should express your answer in terms of y, and Δy. **Include units.**

Solution: We multiply the mass of the slice by g to get its weight, and then multiply by the distance it travels to get the work done on the slice. Therefore, the work done is $\pi(6 - \sqrt{y})^2(3 - 0.1y)(9.8)(16 - y)\Delta y$.

Answer: $9.8\pi(6 - \sqrt{y})^2(3 - 0.1y)(16 - y)\Delta y$ **Units:** Joules

d. [3 points] Write an expression involving one or more integrals representing the work needed to pump all the liquid chemicals to top of the tank, using the same units as in part **c**. **Do not** evaluate any integrals in your expression.

Solution: We note that the lower bound of y should be 0, and the upper bound of y should be 9. Thus the total work done is $\int_0^9 9.8\pi(6 - \sqrt{y})^2(3 - 0.1y)(16 - y)\, dy$ Joules.

Answer: $\int_0^9 9.8\pi(6 - \sqrt{y})^2(3 - 0.1y)(16 - y)\, dy$

10. [12 points]

a. [6 points] For each of the following sequences or series below, determine whether they must converge, must diverge, or whether there is not enough information. Circle your answers. No justification is required.

(i) $a_n = \int_1^n f(x)\,dx$ where $f(x) \geq 0$, $f'(x) \leq 0$, and $\lim\limits_{x\to\infty} f(x) = 0$.

Circle one: **Converges** **Diverges** $\boxed{\textbf{Not Enough Information}}$

(ii) $\sum\limits_{n=1}^{\infty}(-1)^n(1+s^{-n})$ where s is a positive real number.

Circle one: **Converges** $\boxed{\textbf{Diverges}}$ **Not Enough Information**

(iii) $\sum\limits_{n=1}^{\infty} \frac{\sin n}{k^n}$ where k is a real number with $k > e$.

Circle one: $\boxed{\textbf{Converges}}$ **Diverges** **Not Enough Information**

b. [6 points] For each of the following sequences, defined for $n \geq 1$, decide whether the sequence is monotone increasing, monotone decreasing, or not monotone, and whether it is bounded or unbounded. Circle your answers. No justification is required.

(i) $b_n = \frac{(-1)^n}{2n}$

*Circle **all** which apply:*

Monotone Increasing **Monotone Decreasing** $\boxed{\textbf{Not Monotone}}$

$\boxed{\textbf{Bounded}}$ **Unbounded**

(ii) $c_n = e^n \cos\left(\frac{1}{n}\right)$

*Circle **all** which apply:*

$\boxed{\textbf{Monotone Increasing}}$ **Monotone Decreasing** **Not Monotone**

Bounded $\boxed{\textbf{Unbounded}}$

(iii) $d_n = \int_2^{2n} \frac{1}{(x-1)^2}\,dx$

*Circle **all** which apply:*

$\boxed{\textbf{Monotone Increasing}}$ **Monotone Decreasing** **Not Monotone**

$\boxed{\textbf{Bounded}}$ **Unbounded**

9. (6 pts) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. How much volume is enclosed by all the chambers? Assume for simplicity that there are infinitely many chambers. Show your work.

Because the chambers grow by a constant factor each time, they form a geometric series. If each is 20% larger than the previous, then the ratio between them is 1.2. But this is the ratio of the larger divided by the smaller, and we want the opposite, so we get $r = 1/1.2 = 5/6$. This is the ratio by which you have to multiply each volume to get the next smaller volume. The total volume, then, is:

$$9 + 9(\frac{5}{6}) + 9(\frac{5}{6})^2 + 9(\frac{5}{6})^3 \dots.$$

This geometric series sums to $\frac{9}{1-\frac{5}{6}} = 54$. So the total enclosed volume is 54 cubic inches.

By the way, the numbers given in this problem are not simply made up, but are deduced from the size and shape of a large adult chambered nautilus. The number 54 is the approximate volume of a cylinder with height 2 inches and radius 3 inches (a rough approximation to the organism's size and shape).

Where does $5/6$ come from? Notice that one "band" of the chambers takes about 17 chambers, and (by directly measuring the picture), shrinks the organism by a factor of 3, *in length*. Scaling down by a factor of 3 in length is the same as scaling by a factor of 27 in volume, which should leave $54/27 = 2$ cubic inches. Therefore the first 17 chambers take 52 cubic inches. So we have the equations:

$$\frac{a}{1-r} = 54 \text{ and } \frac{a(1-r^{17})}{1-r} = 52.$$

Solving simultaneously gives $a = 9.5 \approx 9$, $r = .82 \approx 5/6$. This is how the problem was written.

4. [13 points] Rafael finds details of another of TimberCorp's logging operations, this time in a forest of redwoods which initially has 50,000 trees. TimberCorp plans to, at the start of each year, cut down 10% of the trees in the forest, and then over the course of the year replant k trees.

a. [5 points] Let R_n be the number of trees in the forest **at the end** of the nth year of the logging operation. Find expressions for R_1 and R_2. Your answers may involve k. **You do not need to simplify your answers**.

Solution:

$$R_1 = (0.9)50000 + k$$
$$R_2 = (0.9)^2(50000) + 0.9k + k$$

b. [5 points] Find a **closed form** expression for R_n. Closed form means your answer should not include ellipses or sigma notation, and should NOT be recursive. **You do not need to simplify your closed form answer.**

Solution:

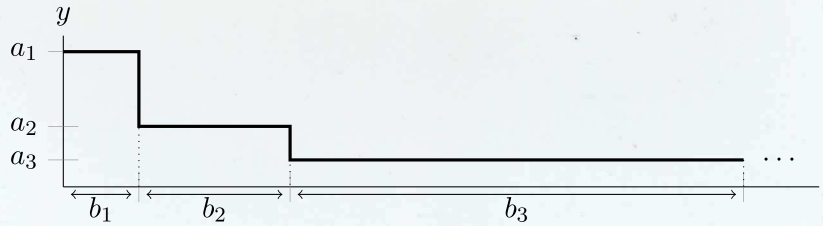
$$\begin{aligned} R_n &= (0.9)^n(50000) + k + 0.9k + (0.9)^2k + \ldots + (0.9)^n k \\ &= (0.9)^n(50000) + k\left(\tfrac{1-(0.9)^n}{1-0.9}\right) \\ &= (0.9)^n(50000) + 10k\,(1-(0.9)^n) \end{aligned}$$

where we used the formula for the sum of a (finite) geometric series

c. [3 points] Rafael wants the number of trees in the forest at the end of a year to tend towards 70,000 in the long run (i.e. after many many years). What value should he choose for k to ensure this happens?

Solution: As $n \to \infty$, $R_n \to 0 + 10k(1-0) = 10k$, and so R_n will tend toward 70,000 if $10k = 70000$, i.e. $k = 7000$.

2. [7 points] The region depicted below consists of infinitely many adjacent rectangles. (Only the first three rectangles are actually shown, and they are not necessarily drawn to scale.) For $n = 1, 2, 3, \ldots$, the nth rectangle has height $a_n = \dfrac{1}{5^{n/2}}$ and width $b_n = n!$.



2

a. [5 points] Write an infinite series that gives the total volume of the solid formed by rotating the entire region (all of the rectangles) around the x-axis.

Each step becomes a cylinder:

$$\text{Volume} = \pi a_1^2 b_1 + \pi a_2^2 b_2 + \cdots$$
$$= \pi \sum_{n=1}^{\infty} a_n^2 b_n = \pi \sum_{n=1}^{\infty} \frac{n!}{(5^{n/2})^2}$$
$$= \pi \sum_{n=1}^{\infty} \frac{n!}{5^n}.$$

b. [2 points] Does the infinite series that gives the total volume of the solid formed by rotating the entire region (all of the rectangles) around the x-axis converge or diverge?

Circle one: **Converges** $\boxed{\textbf{Diverges}}$

State the name of the test you would use to justify your answer. If you would use the comparison test or limit comparison also give a valid comparison series. You do not need to actually write out a full justification. (If you do not know the name of the test you would use, state the test itself.)

RATIO TEST:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{(n+1)!/5^{n+1}}{n!/5^n}$$
$$= \lim_{n\to\infty} \frac{(n+1)!}{n!} \cdot \frac{5^n}{5^{n+1}} = \lim_{n\to\infty} \frac{n+1}{5} = \infty.$$

Since the limit is greater than 1, the series diverges.

7. [14 points] For each of the following sequences

1. Compute $\lim_{n\to\infty} a_n$.
2. Decide if $\sum_{n=0}^{\infty} a_n$ converges or diverges. Circle your answer.

Support your answer by stating the test(s) or facts you used to prove convergence or divergence , and show complete work and justification.

a. [4 points]

$$a_n = \left(\frac{-1}{\pi}\right)^n \qquad \lim_{n\to\infty} a_n = \underline{\hspace{2cm}} \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

Solution: Sequence: $a_n = r^n$ where $|r| = \left|\frac{-1}{\pi}\right| = .318 < 1$ hence $\lim_{n\to\infty} a_n = 0$.

Series: $\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} r^n$ is a geometric series with $|r| < 1$ then it **converges**.

b. [4 points]

$$a_n = \frac{n^2+2}{1+4n^2} \qquad \lim_{n\to\infty} a_n = \underline{\hspace{2cm}} \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

Solution: Sequence: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2+2}{1+4n^2} = \lim_{n\to\infty} \frac{n^2}{4n^2} = \frac{1}{4}$.

Series: Since a_n does not converge to 0 then $\sum_{n=0}^{\infty} a_n$ diverges.

Note to the graders: The criteria for the justification of the divergence of the series has been given different names in some sections (nth term test and some others). If you see these kind of justifications, please ask the instructor before considering any deductions.

c. [6 points]

$$a_n = \frac{n}{\sqrt{n^4+5}} \qquad \lim_{n\to\infty} a_n = \underline{\hspace{2cm}} \qquad \sum_{n=0}^{\infty} a_n : \quad \text{Converges} \quad \text{Diverges}$$

Solution: Sequence: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{\sqrt{n^4+5}} = \lim_{n\to\infty} \frac{n}{n^2} = 0$.

Series: $a_n = \frac{n}{\sqrt{n^4+5}} \sim \frac{1}{n}$ as n goes to infinity.
Limit Comparison Test:

$$\lim_{n\to\infty} \frac{a_n}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{n}{\sqrt{n^4+5}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{\sqrt{n^4+5}} = \lim_{n\to\infty} \frac{n^2}{n^2} = 1.$$

$\sum_{n=0}^{\infty} a_n \sim \sum_{n=0}^{\infty} \frac{1}{n}$. Hence diverges.

5. [6 points] Determine if the following series converges or diverges, and circle the corresponding word. **Fully justify** your answer including using **proper notation** and **showing mechanics** of any tests you use.

$$\sum_{n=1}^{\infty} \frac{4^n}{n!}$$

Circle one: $\boxed{\textbf{Converges}}$ **Diverges**

Justification:

Solution: We use the Ratio Test to determine whether the given series converges or diverges. First, we form

$$\frac{|a_{n+1}|}{|a_n|} = \frac{\frac{4^{n+1}}{(n+1)!}}{\frac{4^n}{n!}} = \frac{4^{n+1}}{4^n} \cdot \frac{n!}{(n+1)!} = \frac{4}{n+1}.$$

Thus

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{4}{n+1} = 0 < 1.$$

Hence, the series $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n!}$ converges by the Ratio test.