

MATH 115 — PRACTICE FOR EXAM 1

Generated October 25, 2026

UMID: SOLUTIONS INITIALS: _____

INSTRUCTOR: _____ SECTION NUMBER: _____

1. This exam has 14 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You may not use a calculator. You are allowed one double-sided 8×11 inch page of handwritten notes.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. You must use the methods learned in this course to solve all problems.
7. You are responsible for reading and following all directions provided for each exam in this course.

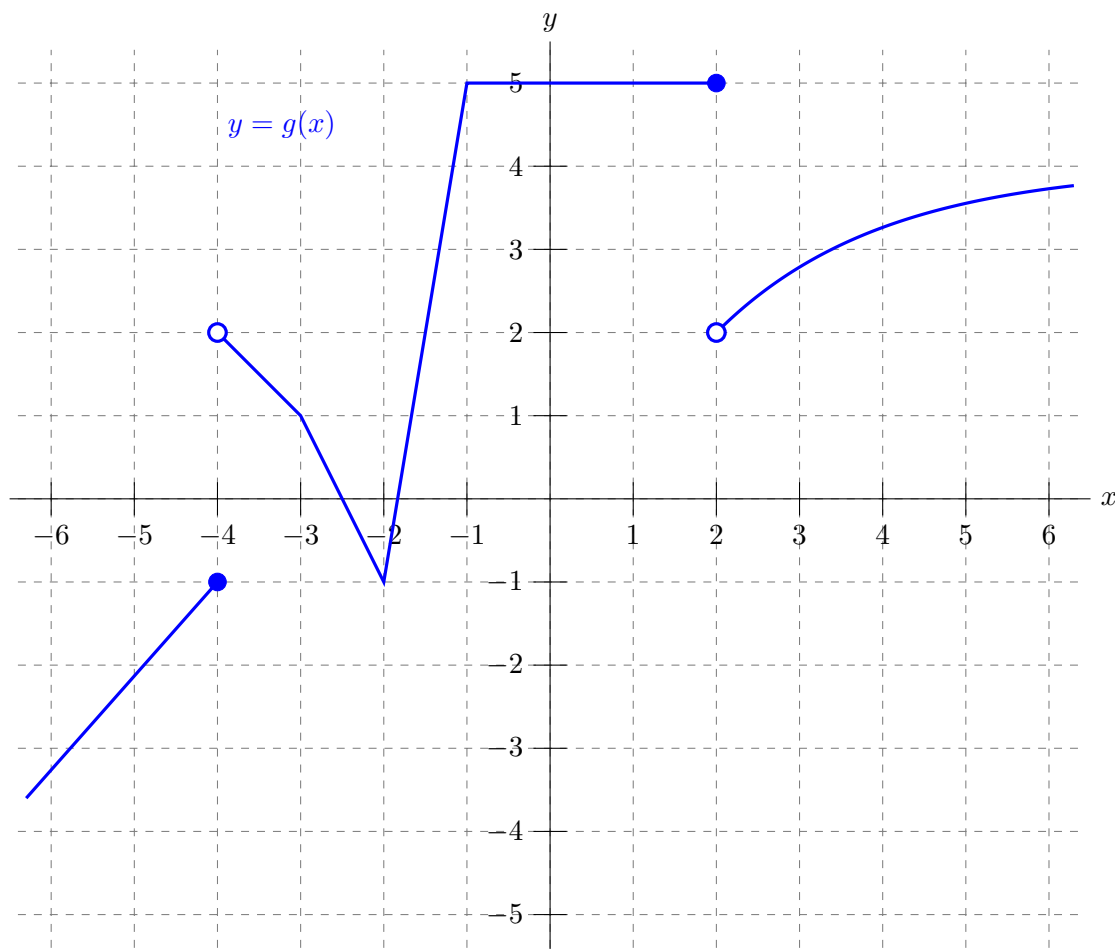
Semester	Exam	Problem	Name	Points	Score
Fall 2016	1	6		12	
Winter 2020	1	7	sheep and cows	11	
Fall 2022	1	5	turbine	8	
Fall 2021	2	3	toy airplane	15	
Fall 2019	1	6		4	
Fall 2019	1	7		9	
Winter 2025	1	8		8	
Fall 2019	2	8	wind chill	11	
Winter 2019	2	9		9	
Fall 2017	2	2		12	
Fall 2022	1	9	train	8	
Fall 2020	1	7		12	
Fall 2020	1	8		11	
Fall 2021	1	8		4	
Total				134	

Recommended time (based on points): 126 minutes

6. [12 points] On the axes provided below, sketch the graph of a single function $y = g(x)$ satisfying all of the following:

- $g(x)$ is defined for all x in the interval $-6 < x < 6$.
- For all x in the interval $-6 < x < -4$, the function $g(x)$ is continuous at x and $g'(x) > 0$.
- $g(-4) = -1$.
- $\lim_{x \rightarrow -4^+} g(x) = 2$.
- $g(-3) = 1$.
- $g(-2) = -1$.
- The function $g(x)$ is continuous on the interval $[-3, -1]$.
- The average rate of change of $g(x)$ between $x = -3$ and $x = -1$ is 2.
- $g'(1) = 0$.
- $g(x)$ is not continuous at $x = 2$.
- The function $g(x)$ is continuous on the interval $3 < x < 6$.
- The slope of the tangent line to the graph of $y = g(x)$ at $x = 3$ is positive.
- $g(x)$ is increasing and concave down on the interval $4 < x < 6$.

Make sure that your graph is large and unambiguous. Note that many solutions are possible.



7. [11 points] Falco decides to raise sheep and cows on his farm.

- Let $W(p)$ be the amount of wool, in pounds per year, produced by a sheep who was fed p pounds of food per day.
- Let $M(p)$ be the amount of milk, in thousands of gallons per year, produced by a cow who was fed p pounds of food per day.

The functions $W(p)$ and $M(p)$ are differentiable and invertible.

a. [2 points] Use a complete sentence to give a practical interpretation of the equation

$$W^{-1}(28) = 10.$$

Solution: A sheep fed 10 pounds of food per day produced 28 pounds of wool per year.

b. [3 points] Write a single equation representing the following statement in terms of the functions W , M , and/or their inverses:

A sheep that produced 12 pounds of wool per year was fed 5 fewer pounds of food per day than a cow that produced 2430 gallons of milk per year.

Answer: $M^{-1}(2.43) = W^{-1}(12) + 5$

c. [3 points] Complete the following sentence to give a practical interpretation of the equation

$$M'(23) = 0.15.$$

If Falco feeds a cow 22.4 pounds of food per day instead of 23 pounds of food per day, then ...

Solution: ...the cow will produce approximately 90 fewer gallons of milk per year.

d. [3 points] Circle the one sentence that gives a valid interpretation of the equation

$$(W^{-1})'(12) = 0.7.$$

- (i.) To increase a sheep's wool production from 12 pounds per year to 12.6 pounds per year, Falco should feed it about 0.42 extra pounds of food per day.
- ii. To increase a sheep's wool production from 12 pounds per year to 12.1 pounds per year, Falco should feed it approximately 0.7 more pounds of food per day.
- iii. A sheep that is fed 12.3 pounds of food per day instead of 12 pounds of food per day will produce approximately 0.21 additional pounds of wool per year.
- iv. When a sheep produces 12 pounds of wool per year, feeding it an extra pound of food per day will cause it to produce about 0.7 additional pounds of wool per year.

5. [8 points] The amount of power produced by a wind turbine depends on the speed of the wind. In particular, suppose $P(s)$ is the power, in megajoules per hour (MJ/h), produced by the turbine when the speed of the wind is s kilometers per hour (km/h). Also suppose that $W(t)$ gives the wind speed, in km/h, at the turbine's location t hours after noon on a certain day.

Assume that $P(s)$ is invertible, and that both $P(s)$ and $W(t)$ are differentiable.

- a. [2 points] Use a complete sentence to give a practical interpretation of the equation

$$P(W(0)) = 8.$$

Solution: At noon, the turbine produces 8 MJ/h of power.

- b. [3 points] Complete the following sentence to give a practical interpretation of the equation

$$W'(4) = 18.$$

From 4:00pm to 4:10pm, ...

Solution: the wind speed at the turbine's location increases by approximately 3 km/h.

- c. [3 points] Circle the one statement below that is best supported by the equation

$$(P^{-1})'(13) = 2.9.$$

- i. When the turbine is generating 13 MJ/h of power, an increase of one km/h in wind speed will produce approximately 2.9 MJ/h more power.
- ii.

If the turbine is producing 13 MJ/h of power, the wind speed must increase by approximately 2.9 km/h to produce an additional MJ/h of power.
- iii. If the wind is blowing at 13 km/h and increases to 14 km/h, the power produced by the turbine will increase by about 2.9 MJ/h.
- iv. If the wind speed is 13 km/h, the power generation of the turbine will increase by one MJ/h if the wind speed increases to about 15.9 km/h.

3. [15 points] Let the differentiable function $h(t)$ represent the height in inches (in) of a toy airplane above the ground at time t seconds (sec). Below is a table of some values for $h(t)$ and $h'(t)$. Assume that $h(t)$ is invertible, and that $h'(t)$ is differentiable for $t > 0$.

t	0	2	4	6	8
$h(t)$	28	19	11	8	4
$h'(t)$	-5	-4	-2	-1.5	-0.5

For parts **a.–d.**, you do not need to show work, but partial credit can be earned from work shown. You do not need to simplify numerical answers.

- a. [3 points] Approximate $h''(8)$. Include units.

Solution:

$$h''(8) \approx \frac{h'(8) - h'(6)}{8 - 6} = \frac{-0.5 - (-1.5)}{2} = \frac{1}{2}.$$

Answer: 0.5 in/sec²

- b. [3 points] Find a formula for the linear approximation $L(t)$ to the function $h(t)$ at $t = 2$.

Solution: $L(t) = h(2) + h'(2)(t - 2) = 19 - 4(t - 2) = -4t + 27$

Answer: $L(t) =$ $19 - 4(t - 2) = -4t + 27$

- c. [2 points] Use your answer from the previous part to approximate $h(1.9)$. Include units.

Solution: $h(1.9) \approx L(1.9) = 19 - 4(1.9 - 2) = 19 - 4(-0.1) = 19.4$

Answer: 19.4 inches

- d. [2 points] Compute the **exact** value of $(h^{-1})'(8)$. (You do not need to include units.)

Solution: $(h^{-1})'(8) = \frac{1}{h'(h^{-1}(8))} = \frac{1}{h'(6)} = \frac{1}{-1.5}.$

Answer: $\frac{1}{-1.5} = -\frac{2}{3}$

- e. [3 points] Suppose that $(h^{-1})'(3) = -9$. Complete the following sentence to give a practical interpretation of this equation.

When the toy airplane is at a height of 3 inches, to descend an additional 0.1 inches ...

Solution: ... will take about 0.9 seconds.

- f. [2 points] Note that $h(t)$ satisfies the hypotheses of the Mean Value Theorem on $[0, 8]$. Complete the following sentence about what the conclusion of this theorem implies is true.

At some time between $t = 0$ and $t = 8$, the height of the toy airplane is ...

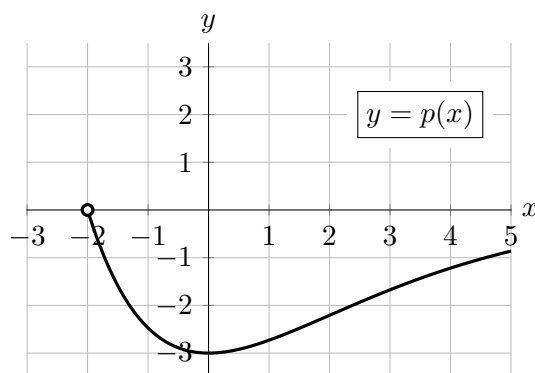
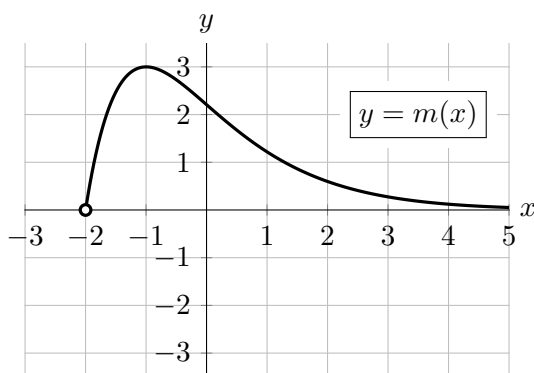
(circle one)

INCREASING

DECREASING

at a rate of 3 in/sec.

6. [4 points] Shown below at left is a portion of the graph of a function $m(x)$. Shown below at right is a portion of the graph of a function $p(x)$, which can be obtained from $m(x)$ through one or more graph transformations. Find a formula for $p(x)$ in terms of $m(x)$.



Answer: $p(x) = -m\left(\frac{1}{2}x - 1\right) = -m\left(\frac{1}{2}(x - 2)\right)$

7. [9 points] For a constant c , let

$$K(x) = \frac{2^{cx}}{e^{x-c}}.$$

- a. [5 points] Use the limit definition of the derivative to write an explicit expression for $K'(3)$. Your answer may include the constant c but should not involve the letter K . Do not attempt to evaluate or simplify the limit. Write your final answer in the answer box provided below.

Answer: $K'(3) =$

$$\lim_{h \rightarrow 0} \frac{\frac{2^{c(3+h)}}{e^{(3+h)-c}} - \frac{2^{c(3)}}{e^{3-c}}}{h}$$

- b. [4 points] Find the value of c so that $K(1) = 5$. Give your answer in **exact form** and show all your work.

Solution: We want c such that

$$\frac{2^{c(1)}}{e^{1-c}} = 5, \text{ or}$$

$$2^c = 5e^{1-c}.$$

Solving, we find that $\ln(2^c) = \ln(5e^{1-c})$

$$\ln(2^c) = \ln(5) + \ln(e^{1-c})$$

$$c \ln(2) = \ln(5) + 1 - c$$

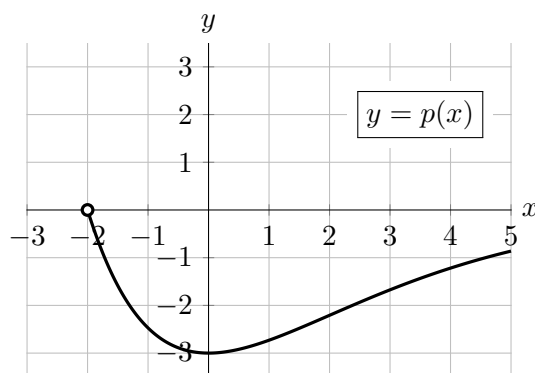
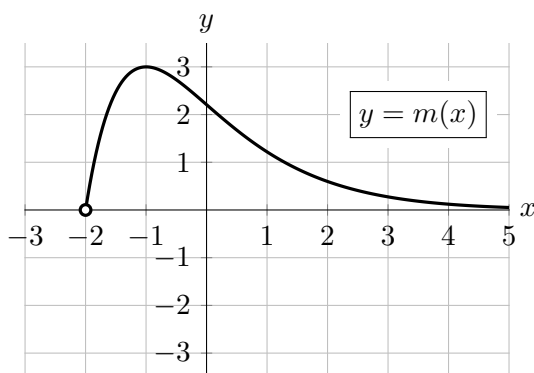
$$c \ln(2) + c = \ln(5) + 1$$

$$c(\ln(2) + 1) = \ln(5) + 1$$

$$c = \frac{\ln(5) + 1}{\ln(2) + 1}.$$

Answer: $c = \frac{\ln(5) + 1}{\ln(2) + 1}$

6. [4 points] Shown below at left is a portion of the graph of a function $m(x)$. Shown below at right is a portion of the graph of a function $p(x)$, which can be obtained from $m(x)$ through one or more graph transformations. Find a formula for $p(x)$ in terms of $m(x)$.



Answer: $p(x) = -m\left(\frac{1}{2}x - 1\right) = -m\left(\frac{1}{2}(x - 2)\right)$

7. [9 points] For a constant c , let

$$K(x) = \frac{2^{cx}}{e^{x-c}}.$$

- a. [5 points] Use the limit definition of the derivative to write an explicit expression for $K'(3)$. Your answer may include the constant c but should not involve the letter K . Do not attempt to evaluate or simplify the limit. Write your final answer in the answer box provided below.

Answer: $K'(3) =$

$$\lim_{h \rightarrow 0} \frac{\frac{2^{c(3+h)}}{e^{(3+h)-c}} - \frac{2^{c(3)}}{e^{3-c}}}{h}$$

- b. [4 points] Find the value of c so that $K(1) = 5$. Give your answer in **exact form** and show all your work.

Solution: We want c such that

$$\frac{2^{c(1)}}{e^{1-c}} = 5, \text{ or}$$

$$2^c = 5e^{1-c}.$$

Solving, we find that $\ln(2^c) = \ln(5e^{1-c})$

$$\ln(2^c) = \ln(5) + \ln(e^{1-c})$$

$$c \ln(2) = \ln(5) + 1 - c$$

$$c \ln(2) + c = \ln(5) + 1$$

$$c(\ln(2) + 1) = \ln(5) + 1$$

$$c = \frac{\ln(5) + 1}{\ln(2) + 1}.$$

Answer: $c = \frac{\ln(5) + 1}{\ln(2) + 1}$

8. [8 points] Sound intensity, or loudness, as measured in decibels, decreases as you move further away from a sound source. If you move from a distance of a meters away from a certain sound source to a new distance of b meters away from it, then the sound intensity changes by

$$20 \log \left(\frac{a}{b} \right) \text{ decibels.}$$

- a. [3 points] Suppose a certain sound source has an intensity of 97 decibels when you stand 9 meters away from it. How far away from the sound source do you need to stand in order to reduce its sound intensity to 90 decibels? *Show all your work, and give your answer in exact form.*

Solution: We are initially standing $a = 9$ meters from the sound source, and need to determine a new distance b meters away from the sound source so that the loudness changes by $90 - 97 = -7$ decibels. So we need to solve the equation

$$20 \log \left(\frac{9}{b} \right) = -7$$

for b . Solving this gives us:

$$\log \left(\frac{9}{b} \right) = -\frac{7}{20}, \quad \frac{9}{b} = 10^{-7/20}, \quad b = \frac{9}{10^{-7/20}} = 9 \cdot 10^{7/20}.$$

Answer: $9/(10^{-7/20}) = 9 \cdot 10^{7/20}$ meters

- b. [3 points] Write a sentence that gives a practical interpretation, in the context of sound intensity, of the following statement:

$$\text{for any positive number } d, \quad 20 \log \left(\frac{d}{10d} \right) = -20.$$

Your sentence should *not* include the variable d .

Solution: Moving 10 times as far away from a sound source decreases its loudness by 20 decibels.

- c. [2 points] Suppose the sound intensity S in decibels at a game in the Big House is given as a function of crowd attendance by $S = f(p)$, where p is the number of fans in attendance. Complete the sentence below to give a practical interpretation of the equation

$$f'(90,000) = 10^{-4}.$$

When there are 90,000 fans in the Big House, you have to add about 10,000 more fans in order to increase crowd noise by 1 decibel.

8. [11 points] Parts **a.** and **b.** are unrelated.

a. [6 points] Windchill is the temperature felt on exposed skin due to the combination of air temperature and wind speed. For a certain fixed air temperature, we define the following functions W and T .

- $W(s)$ is the windchill, in degrees Fahrenheit, when the wind speed is s miles per hour (mph).
- $T(r)$ is the time, in minutes, it takes for frostbite to develop on exposed skin when the windchill is r degrees Fahrenheit.

The functions W and T are both invertible and differentiable. Suppose that

- | | | |
|-------------------|-----------------|--------------------|
| • $W(25) = -37$ | • $T(-25) = 25$ | • $T(-37) = 10$ |
| • $W'(25) = -0.4$ | • $T'(-25) = 2$ | • $T'(-37) = 0.75$ |

i. [2 points] Write an equation for the linear approximation $L(s)$ of $W(s)$ near $s = 25$.

Answer: $L(s) = \underline{\hspace{2cm} -37 - 0.4(s - 25) \hspace{2cm} }$

ii. [1 points] How many minutes does it take for frostbite to develop if the wind speed is 25 mph?

Answer: $\underline{\hspace{2cm} 10 \hspace{2cm} }$

iii. [3 points] If the wind speed is 26 mph, estimate the amount of time, in minutes, it takes for frostbite to develop.

Solution: To estimate $T(W(26))$, one option is to use the linear approximation of $T(W(s))$ near $s = 25$. By the chain rule, $\frac{d}{ds}(T(W(s))) = T'(W(s))W'(s)$, so this linear approximation is

$$\begin{aligned} T(W(25)) + T'(W(25))W'(25)(s - 25) &= 10 + (0.75)(-0.4)(s - 25) \\ &= 10 - 0.3(s - 25). \end{aligned}$$

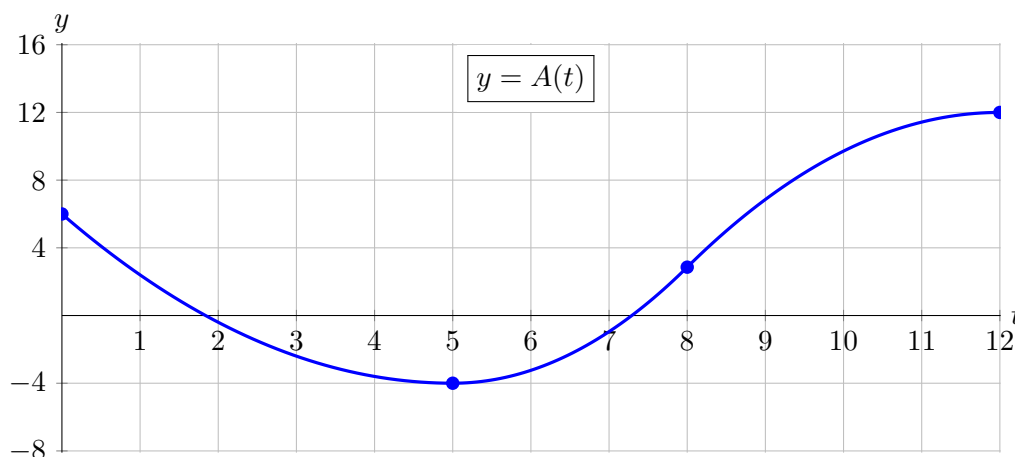
So $T(W(26)) \approx 10 - 0.3(26 - 25) = 9.7$.

Answer: $\underline{\hspace{2cm} 9.7 \hspace{2cm} }$

b. [5 points] Let $A(t)$ be the temperature, in degrees Fahrenheit ($^{\circ}\text{F}$), at time t hours after midnight on a certain winter day in Ann Arbor. You are given the following information.

- $A(t)$ is differentiable and has only one critical point on $0 < t < 12$.
- The coldest temperature that day was -4°F , which occurred at 5:00 AM.
- Between midnight and 5:00 AM, the temperature fell at an average rate of 2°F per hour.
- The temperature was increasing the fastest at 8:00 AM.
- The global maximum value of $A(t)$ on $0 \leq t \leq 12$ is 12°F .

On the axes below, sketch a possible graph of $A(t)$ on $0 \leq t \leq 12$.



9. [9 points] Consider the function

$$f(x) = \begin{cases} -2e^{2x-2} & x \leq 1 \\ x^3 - 3x^2 & x > 1. \end{cases}$$

- a. [5 points] Find all critical point(s) of $f(x)$. Write NONE if there are none.

Solution: The derivative of $y = -2e^{2x-2}$ is $\frac{dy}{dx} = -4e^{2x-2}$. So there are no critical points for $x < 1$. Also, at $x = 1$, this piece has a slope of -4 .

The derivative of $y = x^3 - 3x^2$ is $\frac{dy}{dx} = 3x^2 - 6x = 3x(x - 2)$ which is zero at $x = 0$ and $x = 2$. Only $x = 2$ is on the domain of this piece, so $x = 2$ is a critical point and $x = 0$ is not. Also, at $x = 1$, this piece has a slope of -3 .

Since the slopes aren't equal on the left and right sides of $x = 1$, the function $f(x)$ can't be differentiable there. So $x = 1$ is also a critical point.

Answer: Critical point(s) at $x =$ 1 and 2

- b. [4 points] Find the x -coordinate of all global maxima and global minima of $f(x)$ on the interval $(-\infty, 4]$. For each, write NONE if there are none.

Solution: Since $f(x)$ is continuous (because both pieces equal -2 when $x = 1$), we can compare the value of $f(x)$ at the critical points $x = 1$ and $x = 2$ along with the end point $x = 4$, and we need to consider the end behavior as $x \rightarrow \infty$:

$$f(1) = -2e^{2(1)-2} = -2e^0 = -2$$

$$f(2) = (2)^3 - 3(2)^2 = 8 - 12 = -4$$

$$f(4) = (4)^3 - 3(4)^2 = 64 - 48 = 12$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} -2e^{2x-2} = 0$$

So, $f(x)$ has a global maximum at $x = 4$ and a global minimum at $x = 2$.

Answer: global max(es) at $x =$ 4

Answer: global min(s) at $x =$ 2

2. [12 points] Let $g(x)$ be a continuous function whose first and second derivatives are given below.

$$g'(x) = e^{2x} (2x - 1)^3 (x - 3)^4 \quad \text{and} \quad g''(x) = 4e^{2x} (x^2 - 4) (2x - 1)^2 (x - 3)^3$$

- a. [6 points] Find all values of x at which $g(x)$ has a local extremum. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank below, write NONE if appropriate.

Solution: The critical points of g are when $g'(x) = 0$, so at $x = 1/2$ and $x = 3$. Noticing that $e^{2x} > 0$ for all x , we see that:

- When $x < 1/2$, $g'(x) = (+)(-)(+) = (-)$, so g is decreasing.
- When $1/2 < x < 3$, $g'(x) = (+)(+)(+) = (+)$, so g is increasing.
- When $3 < x$, $g'(x) = (+)(+)(+) = (+)$, so g is increasing.

Therefore, g has a local minimum at $x = 1/2$, and no local maximum.

- b. [6 points] Find all values of x at which $g(x)$ has an inflection point. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points. Write NONE if $g(x)$ has no points of inflection.

Solution: We see that $g''(x) = 0$ when $x = -2, 2, 1/2, 3$. We still need to check whether g changes concavity at each of these points.

- When $x < -2$, $g''(x) = (+)(+)(+)(-) = (-)$, so g is concave down.
- When $-2 < x < 1/2$, $g''(x) = (+)(-)(+)(-) = (+)$, so g is concave up.
- When $1/2 < x < 2$, $g''(x) = (+)(-)(+)(-) = (+)$, so g is concave up.
- When $2 < x < 3$, $g''(x) = (+)(+)(+)(-) = (-)$, so g is concave down.
- When $3 < x$, $g''(x) = (+)(+)(+)(+) = (+)$, so g is concave up.

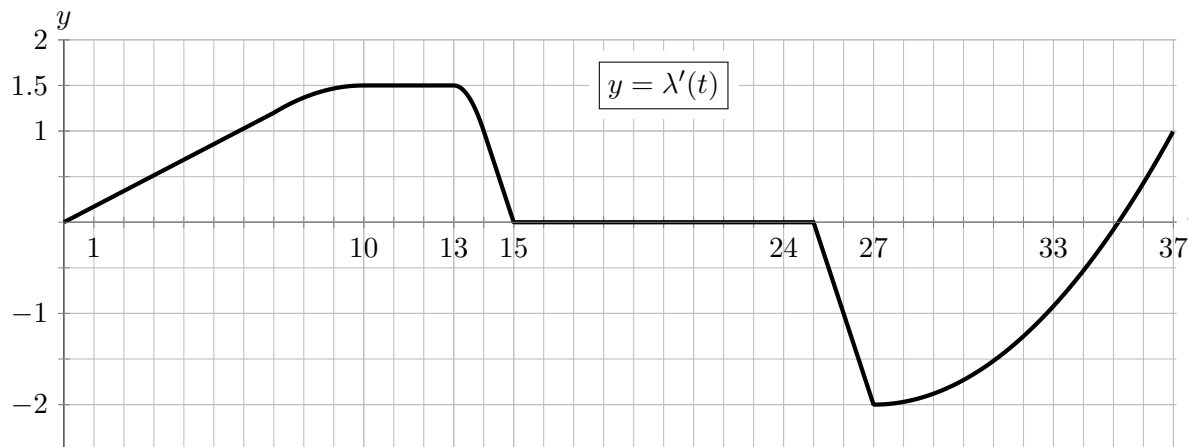
Therefore g has inflection points at $x = -2, 2, 3$.

9. [8 points] The *Lambda* app, developed by your friends, displays information about a train that departed the Detroit station at noon and is traveling on the track between Detroit and Ann Arbor.

The app shows you several values of $\lambda(t)$, the differentiable function that gives the distance along the track, in kilometers, from the Detroit station to the train t minutes after noon:

t	37	39	41	43	45
$\lambda(t)$	16	16	24	30	36

The graph of $\lambda'(t)$, the derivative of $\lambda(t)$, for $0 \leq t \leq 37$, is also shown:



For parts **a.** and **b.**, you may estimate values from the graph as needed.

- a. [1 point] Find all times t for $0 < t < 37$ when the train is traveling at its maximum *velocity*. Give your answer as value(s) and/or interval(s) of t .

Answer: (10, 13)

- b. [1 point] Find all times t for $0 < t < 37$ when the train is traveling at its maximum *speed*. Give your answer as value(s) and/or interval(s) of t .

Answer: 27

- c. [2 points] Find the average velocity of the train between 12:00pm and 12:45pm. Include units.

Answer: $\frac{36}{45} = \frac{4}{5}$ km per minute

- d. [2 points] Estimate the instantaneous velocity of the train at $t = 41$. Include units.

Solution: $\frac{24-16}{41-39} = \frac{8}{2} = 4$, or $\frac{30-24}{43-41} = \frac{6}{2} = 3$, or the average of these, or $\frac{30-16}{43-39} = \frac{14}{4}$ km per minute

Answer: 3.5 km per minute

- e. [2 points] During which of the following time intervals is the train stopped for the entire time? Circle all correct choices.

(3, 5)

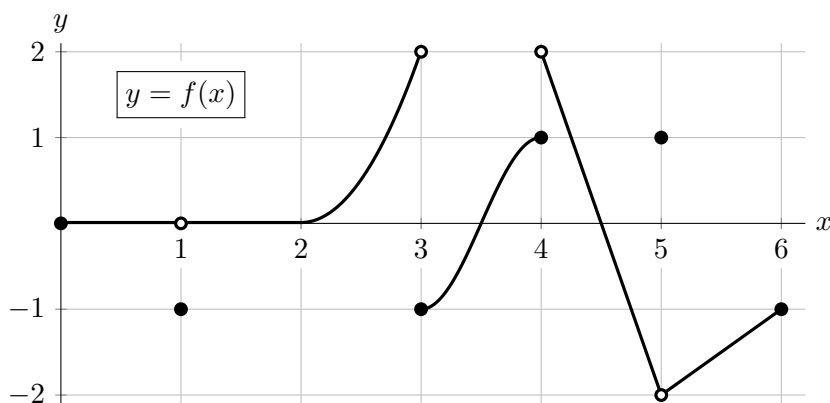
(10, 13)

(18, 21)

(41, 45)

NONE OF THESE

7. [12 points] The graph of the function $f(x)$ is shown below.



For **a.–c.**, give your answers as a list of one or more of the given numbers, or write NONE.

- a. [1 point] At which of the values $a = 1, 2, 3, 4, 5$ is $f(a)$ undefined?

Solution: NONE

- b. [1 point] For which of the values $a = 1, 2, 3, 4, 5$ is $f(x)$ continuous at $x = a$?

Solution: $a = 2$

- c. [2 points] For which of the values $a = 1, 2, 3, 4, 5$ is $f(a) = \lim_{x \rightarrow a^-} f(x)$?

Solution: $a = 2, 4$

For **d.–g.**, use the graph of the function $f(x)$ to evaluate each of the expressions below. If a limit diverges to ∞ or $-\infty$ or if the limit does not exist for any other reason, write DNE.

- d. [2 points] $\lim_{x \rightarrow 5} f(x)$

Solution: Since $\lim_{x \rightarrow 5^-} f(x) = -2$ and $\lim_{x \rightarrow 5^+} f(x) = -2$, the two-sided limit is -2 .

- e. [2 points] $\lim_{x \rightarrow 3} f(x)$

Solution: Since $\lim_{x \rightarrow 3^-} f(x) = 2$ while $\lim_{x \rightarrow 3^+} f(x) = -1$, the two-sided limit does not exist.

- f. [2 points] $\lim_{x \rightarrow 0} f(4 + |x|)$

Solution: As x approaches 0 from both sides, $|x|$ approaches 0 through positive values. So $\lim_{x \rightarrow 0} f(4 + |x|) = \lim_{t \rightarrow 4^+} f(t) = 2$.

- g. [2 points] $\lim_{h \rightarrow 0} \frac{f(4.25 + h) - f(4.25)}{h}$

Solution: This expression represents the slope of the tangent line to the graph of $g(x)$ at $x = 4.25$. On the interval $(4, 5)$ the graph is linear with slope -4 ; so the limit is -4 .

8. [11 points] Consider the rational function $g(x) = \frac{(x-12)(x-7)(x-2)}{(2x-4)(x-3)(x-5)}$.

a. [2 points] What are the vertical asymptotes of the function $g(x)$?

Solution: The vertical asymptotes of $g(x)$ are $x = 3$ and $x = 5$.

(Note that the numerator of $g(x)$ is equal to 0 when $x = 12$, $x = 7$, and $x = 2$, while the denominator is equal to 0 when $x = 2$, $x = 3$, and $x = 5$. Now $g(x) = \frac{(x-12)(x-7)}{2(x-3)(x-5)}$ when $x \neq 2$ so $\lim_{x \rightarrow 2} g(x)$ exists. Therefore $g(x)$ has a “hole” at $x = 2$ rather than a vertical asymptote.)

b. [2 points] What are the vertical asymptotes of the function $\frac{1}{g(x)}$?

Solution: The vertical asymptotes of $\frac{1}{g(x)}$ are $x = 7$ and $x = 12$.

(The function $\frac{1}{g(x)}$ is obtained from the function $g(x)$ by swapping its numerator and denominator. Therefore, the vertical asymptotes of $\frac{1}{g(x)}$ are the zeroes of $g(x)$, namely, $x = 7$ and $x = 12$.)

The piecewise function $h(x)$ is defined as follows, where $g(x)$ is as above, where $f(x)$ is from Problem 7 above, and where B is a nonzero constant.

$$h(x) = \begin{cases} \frac{e^{2x}}{x^2} & x \leq 3 \\ B \cdot f(x) & 3 < x \leq 6 \\ g(x) & 6 < x \end{cases}$$

c. [3 points] Find an *exact* value of B for which the function $h(x)$ is continuous at $x = 3$. Show your work.

Solution: Note that $h(3) = \lim_{x \rightarrow 3^-} h(x) = \frac{e^6}{9}$, while $\lim_{x \rightarrow 3^+} B \cdot f(x) = B \cdot f(3) = -B$. In order for $f(x)$ to be continuous at $x = 3$, these limits must be equal to each other, so $B = -\frac{e^6}{9}$.

Evaluate each of the expressions below. If a limit diverges to ∞ or $-\infty$ or if the limit does not exist for any other reason, write DNE.

d. [2 points] $\lim_{x \rightarrow \infty} h(x)$

Solution: Since $h(x) = g(x)$ when $x > 6$, this limit is the same as $\lim_{x \rightarrow \infty} g(x)$. The limit of this rational function is determined by the leading terms in the numerator and denominator. The leading term of the numerator is x^3 , and the leading term of the denominator is $2x^3$. So

$$\lim_{x \rightarrow \infty} h(x) = \lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} \frac{x^3}{2x^3} = \lim_{x \rightarrow \infty} \frac{1}{2} = \frac{1}{2}.$$

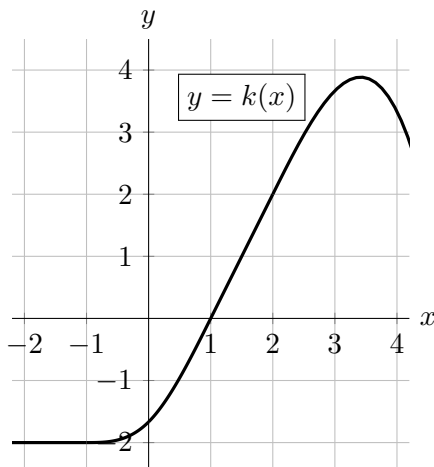
e. [2 points] $\lim_{x \rightarrow -\infty} h(x)$

Solution:

$$\lim_{x \rightarrow -\infty} h(x) = \lim_{x \rightarrow -\infty} \frac{e^{2x}}{x^2} = 0.$$

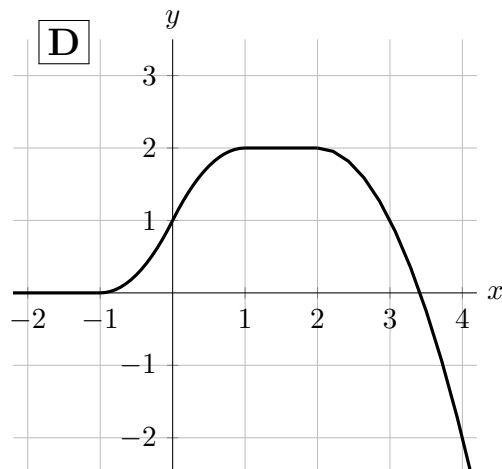
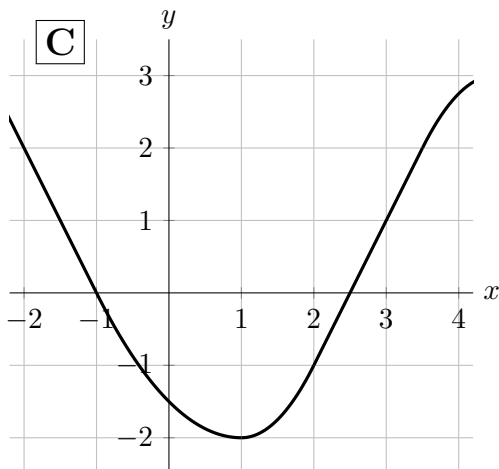
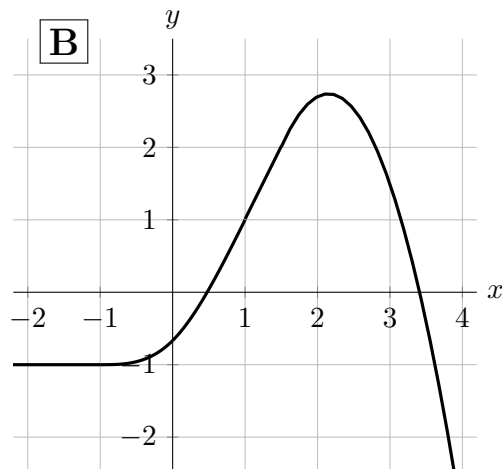
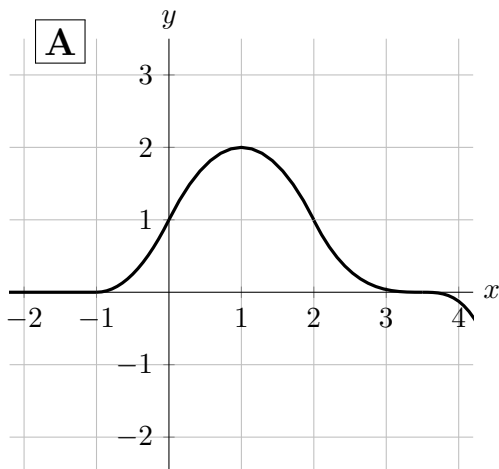
(The first equality holds since $h(x) = \frac{e^{2x}}{x^2}$ when $x \leq 3$. As x grows without bound in the negative direction, e^{2x} approaches 0 while x^2 grows without bound. So $\lim_{x \rightarrow -\infty} \frac{e^{2x}}{x^2} = 0$.)

8. [4 points] A portion of the graph of the differentiable function $k(x)$ is shown below. Note that $k(x)$ is linear on $-2 \leq x \leq -1$ and $1 \leq x \leq 2$.



Which of the following graphs could be the graph of $k'(x)$, the derivative of $k(x)$? There is exactly one correct answer. Write the letter of your answer.

Note: If you are having trouble viewing the graphs, zooming in on the pdf file may help.



Answer: D