Math 116 — Practice for Exam 3

Generated April 26, 2026

UMID: ____ Initials: _____

Instructor: ____________________________ Section Number: __________

---

1. This exam has 16 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You are allowed notes written on two sides of a $3'' \times 5''$ note card. You are NOT allowed other resources, including, but not limited to, notes, calculators or other electronic devices.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. Problems may ask for answers in exact form. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is <u>not</u>.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2019	3	3		10	
Fall 2024	2	11		12	
Winter 2025	3	7		10	
Fall 2024	3	10		9	
Winter 2018	1	5	sandbag	9	
Winter 2025	3	1	boat	13	
Winter 2024	3	2	Maria Luisa	12	
Fall 2018	3	4	butterfly	9	
Fall 2024	3	4		14	
Fall 2025	3	1		9	
Fall 2020	3	1	giraffe	7	
Winter 2025	3	4	chocolate	6	
Winter 2024	2	5		14	
Winter 2022	3	1	movie ratings	13	
Winter 2024	2	4	Plaque Man	11	
Fall 2021	3	9	ice cream	7	
Total				165	

Recommended time (based on points): 184 minutes

3. [10 points] The Taylor series centered at 3 for a function $g(x)$ is given by

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^{2n}}{n\,4^n}.$$

a. [5 points] Determine the radius of convergence for this Taylor series. Show all work.

Radius: ____________________________

b. [2 points] Which of the following best describes the concavity of $g(x)$ at $x = 3$? Circle the one best answer. No justification is necessary.

CONCAVE UP   CONCAVE DOWN   NEITHER   CANNOT BE DETERMINED

c. [3 points] Find $g^{(1010)}(3)$.

$g^{(1010)}(3) =$ ____________________________

11. [12 points]

a. [7 points] Determine the **radius** of convergence for the following power series. Show all of your work. You do not need to find the interval of convergence.

$$\sum_{n=1}^{\infty} \frac{(2n)!}{10^n (n!)(n+1)!} x^{2n}$$

Answer: ______________________

b. [5 points] No justification is needed for the remainder of this problem. Suppose that the following is true about the sequence C_n which is defined for $n \geq 0$:

- C_n is a monotone decreasing sequence of positive numbers which converges to 0.
- $\lim_{n\to\infty} \frac{C_n}{1/n} = 28$.
- The power series $\sum_{n=0}^{\infty} \frac{C_n}{4^n}(x-16)^n$ has radius of convergence 4.

What is the center of the interval of convergence of $\sum_{n=0}^{\infty} \frac{C_n}{4^n}(x-16)^n$?

Answer: ______________________

What are the endpoints of the interval of convergence of $\sum_{n=0}^{\infty} \frac{C_n}{4^n}(x-16)^n$?

Answer: Left endpoint at $c =$ ______________________

Right endpoint at $d =$ ______________________

Let c and d be the left and right endpoints of the interval of convergence you found above. Which of the following could be the interval of convergence of $\sum_{n=0}^{\infty} \frac{C_n}{4^n}(x-16)^n$? Circle **all** correct answers.

(c, d) $\qquad$ $(c, d]$ $\qquad$ $[c, d)$ $\qquad$ $[c, d]$

7. [10 points] Consider the function

$$g(x) = \frac{1}{3}\cos(x^2) - x\sin(x).$$

a. [5 points] Give the first three non-zero terms of the Taylor series of $g(x)$ centered about $x = 0$. Show all your work.

Answer: ______________________________

b. [5 points] The function $g(x)$ has a continuous antiderivative, $G(x)$, with a Taylor series that converges for all x. Given that $G(0) = 8$, find the first four non-zero terms of the Taylor series for $G(x)$ centered about $x = 0$. Show all your work.

Answer: ______________________________

10. [9 points] The parts of this problem are unrelated. No justification is required for your answers.

a. [3 points] Which of the following could be the value of a if

$$1 - \frac{4a^2}{2!} + \frac{16a^4}{4!} - \frac{(2a)^6}{6!} + \frac{(2a)^8}{8!} - \cdots = \frac{1}{2}?$$

Circle **all** options which apply.

i. $a = 0$

ii. $a = \frac{\pi}{6}$

iii. $a = \frac{\pi}{3}$

iv. $a = \frac{\pi}{2}$

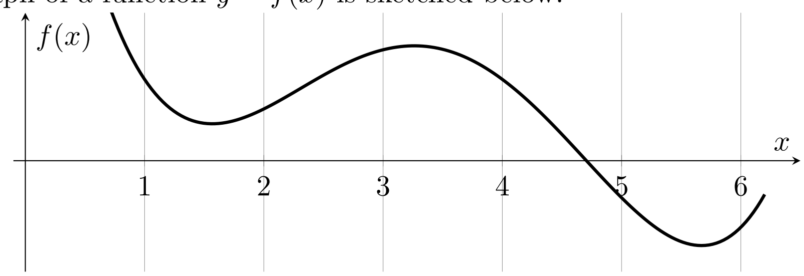
v. $a = \frac{2\pi}{3}$

vi. $a = \frac{5\pi}{6}$

vii. $a = \pi$

viii. NONE OF THESE

b. [3 points] A graph of a function $y = f(x)$ is sketched below.



Suppose that for some constant b, the Taylor polynomial of degree 3 for $f(x)$ around $x = b$ is given by $P_3(x) = 4 - (x - b) + 2(x - b)^2 - 3(x - b)^3$. Which of the following could be the value of b? Circle **all** options which apply.

i. $b = 1$

ii. $b = 2$

iii. $b = 3$

iv. $b = 4$

v. $b = 5$

vi. $b = 6$

c. [3 points] Which of the following is the Taylor series approximation around $x = 0$ to

$$\int_0^x e^{t^2}\, dt?$$

Circle the **one** best option.

i. 0

ii. $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

iii. $\sum_{n=0}^{\infty} \frac{t^{2n}}{n!}$

iv. $\sum_{n=0}^{\infty} \frac{x^n}{2(n!)}$

v. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$

vi. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!(2n+1)}$

5. [9 points] Tammy Toppel is directing a performance art piece at the community center. She fills a large cone with sand and cuts a small hole in the bottom. Gerd Hömf was hired from a temp agency to stand behind the scenes and steadily lift the cone with an elaborate pulley system, letting the sand slowly spill onto the stage.

a. [2 points] The filled cone starts with a total mass of 40 kilograms and spills sand at a constant rate of 1/2 a kilogram per second once it is lifted. Tammy wants Gerd to lift the cone at a constant rate of r meters per second. Find a formula for the mass $M(h)$, in kilograms, of the cone when it is h meters above the stage.

Answer: $M(h) =$ ____________________

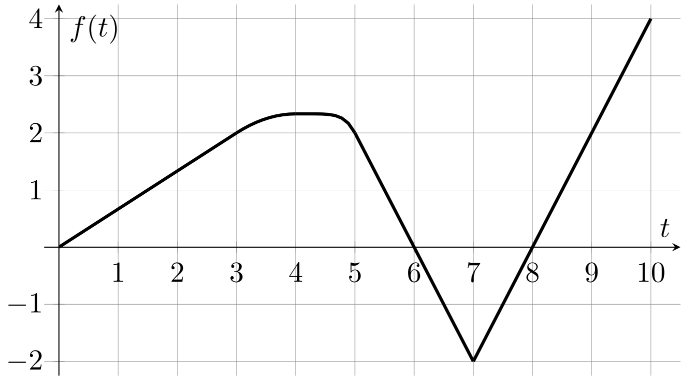
b. [4 points] Gerd lifts the cone until it reaches a height of 20 meters above the stage. Write an integral which represents the work (measured in Joules) done by Gerd while lifting the cone. The integral may include the rate r at which Gerd lifts and g the acceleration (in m/s^2) due to gravity.

Answer: ____________________

c. [3 points] There's one catch: Gerd's contract strictly prohibits him from exerting more than $500g$ Joules of work, where g is the acceleration due to gravity. At what rate r (in m/s) should Tammy ask Gerd to lift in order to not violate his contract and to get the cone lifted as quickly as possible?

Answer: $r =$ ____________________

1. [13 points] Caroline uses a remote-controlled boat to survey a reservoir. The boat starts at the point $(x, y) = (0, 0)$, and after t seconds is positioned at $x = f(t)$ and $y = g(t)$. A graph of $f(t)$ and a formula for $g(t)$ are given below. Note that $f(t)$ is linear on the the intervals $[0, 3]$, $[5, 7]$, and $[7, 10]$, and has a local maximum at $t = 4$.



$$g(t) = 12\cos\left(\frac{\pi}{2}t\right) - 12$$

For each of the following parts, your final answer should **not** include the letters f or g.

a. [2 points] Where is the boat located after 10 seconds?

Answer: $x =$ ______________ and $y =$ ______________

b. [3 points] Are there any times during these 10 seconds at which the boat comes to a complete stop? If so, list all such times. If not, write NONE.

Answer: $t =$ ______________

c. [4 points] Write an expression involving one or more integrals for the total distance traveled by the boat during the **first 3 seconds**. Do not evaluate any integrals in your answer.

Answer: ______________

d. [4 points] What is the tangent line to the boat's path at $t = 9$? Give your answer in cartesian form.

Answer: $y =$ ______________

2. [12 points] In the video game *Super Maria 64*, sisters Maria and Luisa travel through the Lilypad Kingdom to collect magical Rainbow Crystals. In the Sandland Desert, represented by the xy-plane, the sisters run around collecting all the Rainbow Crystals they see.
All distances in this problem are measured in kilometers. For $t \geq 0$, the sisters' positions t hours after they start running are given by the following parametric equations:

$$\text{Maria: } \begin{cases} x(t) = t^2 + t - 6 \\ y(t) = 2\sin(\pi t) \end{cases} \qquad \text{Luisa: } \begin{cases} x(t) = 2t^2 - 4t \\ y(t) = \cos(\frac{\pi}{2}t) \end{cases}$$

a. [2 points] Find **Maria's position** 1 hour after the sisters start running.

Answer: $x =$ ______________ $y =$ ______________

b. [3 points] Find **Maria's speed,** in kilometers per hour, 1 hour after the sisters start running.

Answer: ______________________________

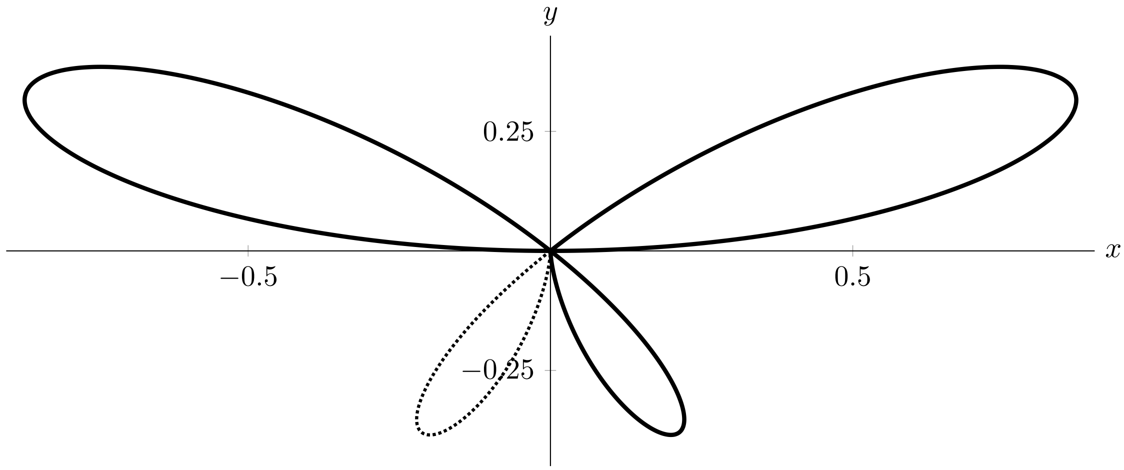
c. [3 points] Find **all** times $t \geq 0$ at which **Luisa** travels **directly north** (that is, not in any northwest or northeast direction). If there is no such time, write "NONE." Show your work to justify your answer.

Answer: $t =$ ______________________

d. [4 points] Find **all** times $t \geq 0$ at which Maria and Luisa are at the same position. If there is no such time, write "NONE." Show your work to justify your answer.

Answer: $t =$ ______________________

4. [9 points] The polar curve $r = \sin(4\theta)\cos(\theta)$ for $0 \leq \theta \leq \pi$ is shown below.



Note that there are two "large loops" and two "small loops".

For reference, note that for this curve, $\dfrac{dr}{d\theta} = 4\cos(\theta)\cos(4\theta) - \sin(\theta)\sin(4\theta)$

a. [3 points] For what values of θ does the polar curve $r = \sin(4\theta)\cos(\theta)$ trace once around the "small loop" in the third quadrant? (This portion of the curve is indicated by the dotted line.) Give your answer as an interval of θ values between 0 and π.

Answer: ________________________________

b. [3 points] Write, but do <u>not</u> evaluate, an expression involving one or more integrals that gives the total arc length of the two small loops.

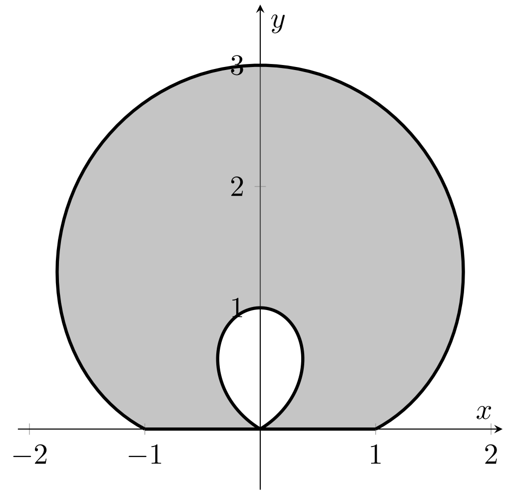
Answer: Arc Length = ________________________________

c. [3 points] Write, but do <u>not</u> evaluate, an expression involving one or more integrals that gives the area of the region that is enclosed by the polar curve $r = 2$ but is outside the curve $r = \sin(4\theta)\cos(\theta)$.

Answer: Area = ________________________________

4. [14 points]

Delema Inventions Inc is coming up with a design for a window. The window has the shape shown to the right, formed from the portion of the polar curve $r(\theta) = 1+2\sin\theta$ with $y \geq 0$.
The outer loop (shaded) is made of green glass, and the inner loop (unshaded) is made of blue glass. The perimeter (including the perimeter of the inner loop, and the base along the x-axis) is lined with a black material.



a. [4 points] There are four values of θ with $0 \leq \theta < 2\pi$ such that $y = 0$ for the polar curve $r(\theta) = 1 + 2\sin\theta$. Find all four values.

Answer: $\theta =$ ____________________________

b. [5 points] Find an expression involving one or more integrals for the length of black material Delema Inventions Inc will need to build the window. Remember that this material lines the edge of the outer loop, the edge of the inner loop, and also the base along the x-axis. Do not evaluate your integral(s).

Answer: ____________________________

c. [5 points] Find an expression involving one or more integrals for the area of green glass (the shaded region) which Delema Inventions Inc will need to build the window. Do not evaluate your integral(s).

Answer: ____________________________

1. [9 points] Some values of the function $g(x)$ and its derivative are given in the table below. Suppose that both $g(x)$ and $g'(x)$ are continuous.

x	1	3	5	7	9
$g(x)$	3	1	4	2	5
$g'(x)$	-2	0	3	-1	4

Using the information given above, find the following. Be sure to **show all of your work.** Your answers should not involve the letter g, but you **do not need to simplify them.**

a. [3 points] Suppose $F(x) = \int_1^{x^2} g(t)\, dt$. Find $F'(3)$.

Answer: ____________________

b. [3 points] Find $\lim_{x\to 1} \dfrac{3\ln(x) + g(x) - 3}{x - 1}$.

Answer: ____________________

c. [3 points] Use MID(2) to find the approximate value of $\int_1^9 \dfrac{g(x)}{1+x^3}\, dx$. Write out all the terms in your sum and do not attempt to simplify.

Answer: ____________________

1. [7 points] At a wildlife sanctuary, Diego fills the giraffes' water bowl at a constant rate of 0.5 gallons per minute. The rate in gallons per minute at which the giraffes drink from the bowl, t **minutes** after 8am, is given by $r(t)$. Suppose there are 12 gallons of water in the bowl at 10am.

a. [3 points] Write an expression possibly involving one or more integrals for the amount of water, in gallons, the giraffes drink between 9am and noon.

b. [4 points] Write an expression possibly involving one or more integrals for the amount of water, in gallons, in the bowl at 8am.

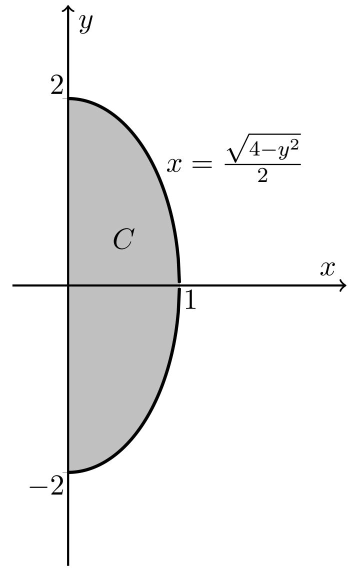
4. [6 points]

Mike owns Mike's Sweet Haven, a bakery that specializes in elegant, custom-made baked goods. With summer approaching, he decides to try something new.

He plans to create a new type of chocolate using the shaded region C, which is bounded by the curves

$$x = \frac{\sqrt{4-y^2}}{2}, \text{ and } x = 0.$$

as illustrated to the right.



a. [4 points] Write an expression involving one or more integrals for the volume of the chocolate obtained by revolving the region C about the y-axis. **Do not** evaluate any integrals in your expression.

Answer: ______________________________

b. [2 points] If Mike were to take a TRAP(4) estimate of the integral you obtained in part **a.**, would he get an underestimate or an overestimate of the volume of the chocolate? No justification is required.

Circle one: **Underestimate** **Overestimate**

5. [14 points]

a. [7 points] Determine whether the following improper integral is convergent or divergent. **Fully justify** your answer including using **proper notation** and **showing mechanics** of any tests you use. You do not need to compute the value of the integral if it is convergent. Circle your final answer choice.

$$\int_1^{\infty} \frac{4+\sin(x)}{x^3+2}\,dx$$

Circle one: **Convergent** **Divergent**

b. [7 points] Let $0 < p < 1$ be a real number, and consider the improper integral

$$\int_1^{3} \frac{1}{t(\ln(t))^p}\,dt.$$

The integral above converges; to show this, **compute** its value. Your answer may involve p. Be sure to show your full computation, and be sure to use **proper notation.** Remember: $0 < p < 1$.

Answer: $\int_1^{3} \frac{1}{t(\ln(t))^p}\,dt =$ ______________________________

1. [13 points] Universe of Movies (UofM) is a new online movie rating database, which assigns movies a rating from zero to five stars. The star ratings for each movie are not necessarily integer values. The following **probability density function** gives the distribution of star rating, t, for the films in the UofM database.

$$p(t) = \begin{cases} 0 & t \leq 0 \\ \frac{t}{6} & 0 < t \leq 3 \\ \frac{t}{32} & 3 < t \leq 5 \\ 0 & 5 < t \end{cases}$$

a. [6 points] Write a formula for the corresponding cumulative distribution function, $F(t)$.

$$F(t) = \begin{cases} \_\_\_\_\_\_\_\_\_\_ & t \leq 0 \\ \_\_\_\_\_\_\_\_\_\_ & 0 < t \leq 3 \\ \_\_\_\_\_\_\_\_\_\_ & 3 < t \leq 5 \\ \_\_\_\_\_\_\_\_\_\_ & 5 < t \end{cases}$$

b. [3 points] What is the median number of stars for the films in the online database? Be sure to show any work.

Answer: ________________

c. [4 points] Write, but do not compute, the formula for the mean number of stars for the movies in the UofM database. Write out formulas of all functions you use, i.e. do not include p or F in your answers.

Answer: ________________

4. [11 points] Zach is playing the retro video game *Plaque-Man* all day to get a new personal high score. Zach starts playing the game with 0 points. Over the course of each hour, Zach scores an additional 2500 points. At the **beginning** of every hour, Zach trades 20% of his points to buy extra time. For $n \geq 1$, let H_n be Zach's score at the **end** of the nth hour of playing the game.
For example, $H_1 = 2500$.

a. [4 points] Write expressions for H_2 and H_3. Your answers should not involve the letter H. You do not need to simplify your expressions.

$H_2 =$ ________________________

$H_3 =$ ________________________

b. [4 points] Write a **closed-form** expression for H_n. *Closed-form* means your answer should not include ellipses ($\ldots$) or sigma notation (Σ), and should not be recursive. You do not need to simplify your closed-form expression.

Answer: $H_n =$ ________________________

c. [3 points] Find Zach's eventual score if he keeps playing *Plaque-Man* indefinitely. You do not need to simplify your numerical answer.

Answer: ________________________

9. [7 points] Emily is transporting a chocolate ice cream cone up 1 story of East Hall to her friend. However, there is a hole in the bottom of the cone and ice cream drips out in a steady stream.

The mass of the cone is 200 grams and there are initially 100 grams of ice cream in the cone. The ice cream drips out at a rate of 4 grams/sec. Emily spends 10 seconds raising the cone at a constant rate of 0.5 m/sec to reach her friend.

a. [3 points] What is the total mass in grams of the cone and the ice cream in the cone when Emily has lifted it a vertical distance ℓ m?

Mass = ____ grams

b. [4 points] Write, but do not evaluate, an integral that represents the total amount of work (in grams m^2/sec^2) done by Emily lifting the cone filled while the ice cream drips. You may assume the acceleration due to gravity is $g = 9.8$ m/sec^2.

Work = ____ grams m^2/sec^2