MATH 115 — PRACTICE FOR EXAM 1

Generated October 15, 2025

UMID: ______________________________ INITIALS: ______

INSTRUCTOR: ______________________ SECTION NUMBER: __________

1. This exam has 11 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You may not use a calculator. You are allowed one double-sided 8×11 inch page of handwritten notes.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. You must use the methods learned in this course to solve all problems.
7. You are responsible for reading and following all directions provided for each exam in this course.

Semester	Exam	Problem	Name	Points	Score
Fall 2023	1	5	birds	7	
Winter 2020	1	4		10	
Fall 2023	1	8	spaceship	8	
Fall 2023	1	4		5	
Winter 2022	1	8	cat cafe	6	
Winter 2019	1	7	infections	12	
Winter 2025	2	2		10	
Winter 2017	2	8	apiary	8	
Fall 2015	1	1		5	
Winter 2022	2	5		15	
Winter 2024	1	1		9	
Total				95	

Recommended time (based on points): 93 minutes

5. [7 points] Since the start of spring, bird enthusiasts Charlie and Parker have been seeing, and feeding, more and more birds in their backyard. Eventually they decide to model the number of birds they see and the amount of birdseed they go through using increasing functions.

Let $B(t)$ be the number of birds they see on the t^{th} day after the start of spring. They start recording values on the third day of spring, and their initial data appear in the table to the right.

t	3	4
$B(t)$	16	20

a. [1 point] Charlie thinks $B(t)$ should be a linear function. Find an expression for $B(t)$ if it is a linear function.

Answer: $B(t) =$ ____________________

b. [2 points] Parker thinks $B(t)$ should be exponential. Find an expression for $B(t)$ if it is an exponential function.

Answer: $B(t) =$ ____________________

After arguing whether $B(t)$ is linear or exponential for (arguably) too long, and wasting a day in the process, they realize that taking a third measurement might settle the debate.

c. [2 points] Based on the additional data shown to the left, circle the one best answer:

t	3	4	6
$B(t)$	16	20	25

(i) $B(t)$ is linear but not exponential

(ii) $B(t)$ is exponential but not linear

(iii) $B(t)$ is both linear and exponential

(iv) $B(t)$ is neither linear nor exponential

Now let $S(t)$ be the amount of birdseed Charlie and Parker use, in pounds, on the t^{th} day after the start of spring.

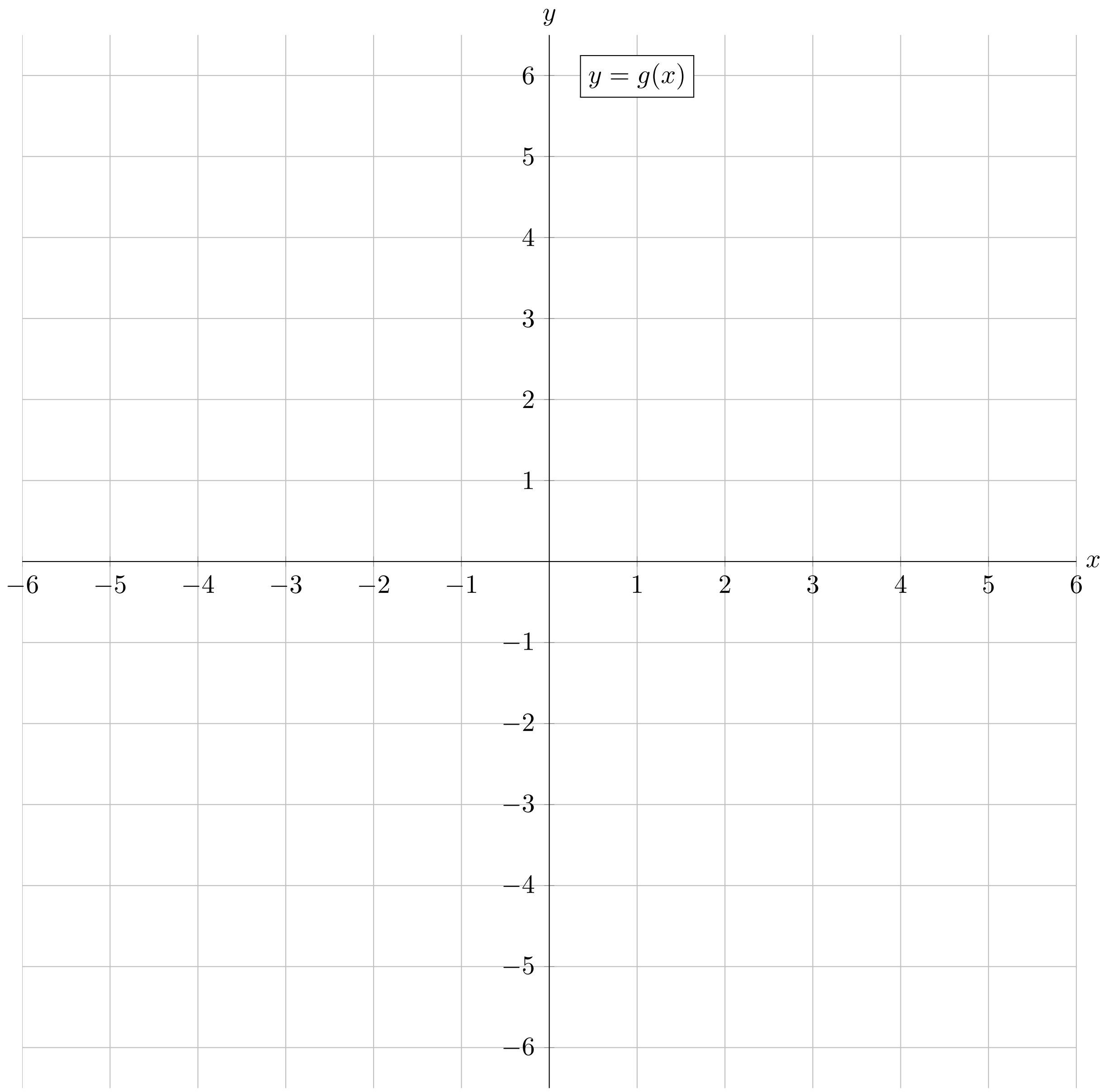
d. [2 points] Write an equation involving B, S, and/or their inverses that represents the following statement:

Charlie and Parker use 2.3 *pounds of birdseed two days after they see* 43 *birds.*

Answer: ____________________

4. [10 points] On the axes provided below, sketch the graph of a single function $g(x)$ that satisfies all of the following conditions:

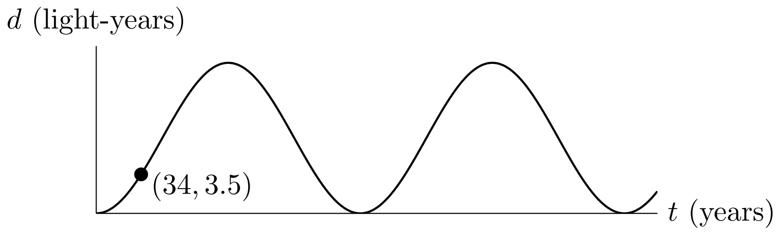
- the domain of the function $g(x)$ contains $-6 < x < 6$
- $g(x)$ is increasing for $-5 < x < -2$
- $\lim_{x \to -2} g(x) = 1$
- $g(x)$ is <u>not</u> continuous at -2
- $g(0) = -3$
- the average rate of change of $g(x)$ from $x = -2$ to $x = 0$ is $-\frac{1}{2}$
- $g(x)$ is constant for $0 < x < 3$
- $\lim_{x \to 4^-} g(x) = g(4)$
- $g(x)$ is <u>not</u> continuous at 4
- $g'(x)$ is constant for $4 < x < 6$



8. [8 points] Far in the future, a particular spaceship transports humans back and forth along a fixed path between the two star systems Alpha and Beta. The distance d in light-years from Alpha to the spaceship t years after its initial departure from Alpha is given by

$$d \;=\; s(t) \;=\; 6.75 - 6.75\cos\left(\tfrac{2\pi}{200}t\right).$$

A graph of $s(t)$, along with a table of a few values of both $s(t)$ and $s'(t)$, is given below.



t	0	9	12	34
$s(t)$	0	0.27	0.47	3.50
$s'(t)$	0	0.06	??	0.19

a. [1 point] How many years does it take for the spaceship to travel from Alpha to Beta?

Answer: ____________________ years.

b. [1 point] How many light-years apart are Alpha and Beta?

Answer: ____________________ light-years.

c. [1 point] Using the table, give the best possible estimate of $s'(12)$.

Answer: ____ light-years per year.

There is a "Mystery Spot" along the spaceship's path where gravity seems to be a little different. The ship first passes this spot 34 years after departing Alpha. (See the graph above.)

d. [3 points] A baby tortoise is born on the spaceship just as it departs Beta, and becomes the ship's mascot, remaining aboard for the rest of its life. Assuming the tortoise lives 212 years, how many times will it get to see the Mystery Spot, and at what age(s)?

Answer: ____________________ times, at ages ______________________________.

e. [2 points] Now suppose the Mystery Spot wrecks the ship's engines at time $t = 34$, so the spaceship is left to drift along forever at the speed and in direction it was going when it reached the Mystery Spot. Under this new assumption, how far away would the spaceship be from Alpha two years after it reached the Mystery Spot? *(Include units. Note that $t = 34$ appears in the graph and in the table.)*

Answer: ____

3. [4 points] Find positive constants a, b, and c such that the function

$$f(x) = \begin{cases} \ln(ce - x) & x \leq 0 \\ \dfrac{ax^3 + \pi}{x^b + 1} & x > 0 \end{cases}$$

is continuous and satisfies $\lim_{x \to \infty} f(x) = 4$. Show your work, and write your answers in exact form.

Answers: $a =$ __________ $b =$ __________ $c =$ __________

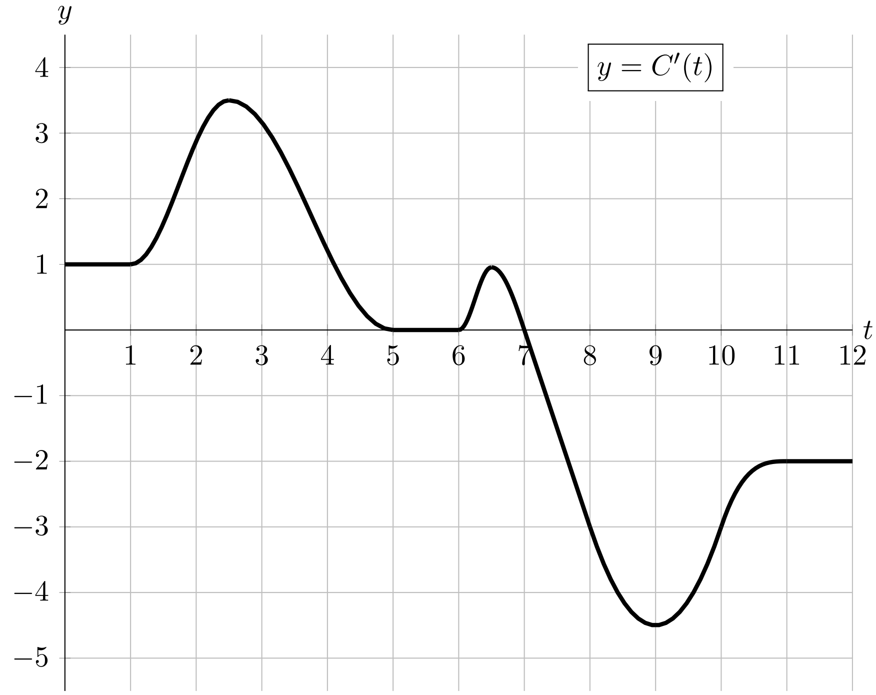
4. [5 points] Let

$$Q(w) = w^w + \cos(6w - 1).$$

Use the limit definition of the derivative to write an explicit expression for $Q'(3)$. *Your answer should not involve the letter Q. Do not attempt to evaluate or simplify the limit.* Write your final answer in the answer box provided below.

Answer: $Q'(3) =$

8. [6 points] Let $C(t)$ be the temperature, in degrees Fahrenheit (°F), of a cat café t hours after noon on a certain winter day. The function $C'(t)$, the **derivative** of $C(t)$, is graphed below.



a. [2 points] Over which of the following intervals of t, if any, is the temperature of the cat café constant? Circle **all** correct answers.

$[0, 1]$ $[5, 6]$ $[7, 8]$ $[11, 12]$ NONE OF THESE

b. [2 points] Over which of the following intervals of t, if any, is the temperature of the cat café decreasing? Circle **all** correct answers.

$[2, 3]$ $[3, 4]$ $[8, 9]$ $[9, 10]$ NONE OF THESE

c. [1 point] At which of the following times t is the temperature in the cat café changing most rapidly? Circle the **one** correct answer.

$t = 1.5$ $t = 2.5$ $t = 8$ $t = 9$

d. [1 point] At which of the following times t is the temperature in the cat café the highest? Circle the **one** correct answer.

$t = 0$ $t = 2.5$ $t = 7$ $t = 12$

7. [12 points]
The annual number of respiratory infections in a city is a function of the amount of carbon in the atmosphere above that city.

Let $R(p)$ be the annual number of respiratory infections in Ann Arbor when there are p thousand tons of carbon in the atmosphere above the city.

Let $C(k)$ be the healthcare cost, in thousands of dollars, of treating k respiratory infections.

The functions $R(p)$ and $C(k)$ are differentiable and invertible.

a. [3 points] Give a practical interpretation of the equation $R^{-1}(212) = 24$.

b. [3 points] Give a practical interpretation of the equation $C(R(17)) = 650$.

c. [3 points] Write a mathematical equation that represents the following statement:

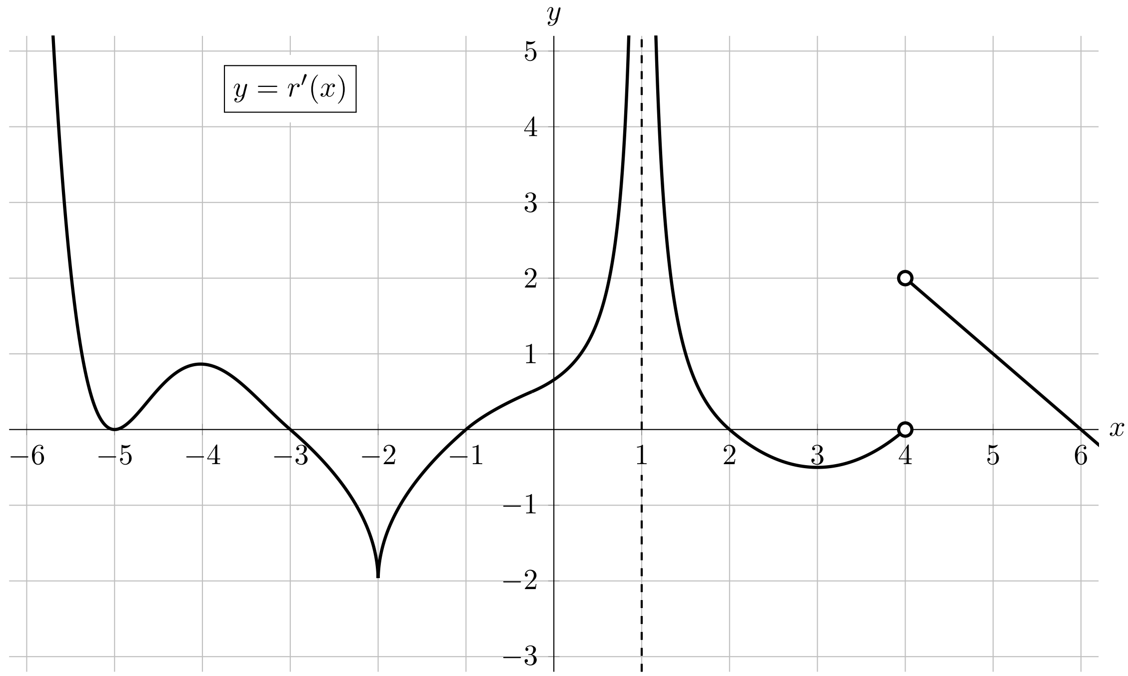
The healthcare cost of treating 165 respiratory infections is 400 thousand dollars more than the healthcare cost of treating 130 respiratory infections.

Answer: ________________________

d. [3 points] Complete the following sentence using the fact that $R'(38) = 4$:

If the amount of carbon in the atmosphere above Ann Arbor is reduced from 41 *thousand tons to* 38 *thousand tons, . . .*

2. [10 points] Suppose $r(x)$ is a continuous function, defined for all real numbers. A portion of the graph of $r'(x)$, the **derivative** of $r(x)$, is given below. Note that $r'(x)$ has a vertical asymptote at $x = 1$ and a sharp corner at $x = -2$, and is undefined only at $x = 1$ and $x = 4$.



a. [2 points] Circle all points below that are critical points of $r(x)$.

$x = -5$ $\quad$ $x = -3$ $\quad$ $x = -2$ $\quad$ $x = 1$ $\quad$ $x = 3$ $\quad$ NONE OF THESE

b. [2 points] Circle all points below that are local maxima of $r(x)$.

$x = -5$ $\quad$ $x = -3$ $\quad$ $x = -1$ $\quad$ $x = 1$ $\quad$ $x = 4$ $\quad$ NONE OF THESE

c. [2 points] Circle all points below that are local minima of $r(x)$.

$x = -5$ $\quad$ $x = -3$ $\quad$ $x = -1$ $\quad$ $x = 1$ $\quad$ $x = 4$ $\quad$ NONE OF THESE

d. [2 points] Circle all points below that are inflection points of $r(x)$.

$x = -5$ $\quad$ $x = -4$ $\quad$ $x = -2$ $\quad$ $x = 2$ $\quad$ $x = 4$ $\quad$ NONE OF THESE

e. [2 points] Circle all intervals below on which $r'(x)$ satisfies the hypotheses of the Mean Value Theorem.

$[-5, -3]$ $\quad$ $[-3, -1]$ $\quad$ $[-2, 0]$ $\quad$ $[0, 2]$ $\quad$ $[2, 4]$ $\quad$ NONE OF THESE

8. [8 points] At Happy Hives Bee Farm, the population of bees, in thousands, t months after the farm opens, can be modeled by $g(t)$, where

$$g(t) = \begin{cases} 20 + \frac{1}{3}e^{4-t} & \text{for } 0 \le t \le 4 \\ \\ -\dfrac{1}{6}t^3 + \dfrac{9}{4}t^2 - 7t + 23 & \text{for } 4 < t \le 8 \end{cases}$$

and

$$g'(t) = \begin{cases} -\frac{1}{3}e^{4-t} & \text{for } 0 < t < 4 \\ -0.5(t-2)(t-7) & \text{for } 4 < t < 8. \end{cases}$$

a. [6 points] Find the values of t that minimize and maximize $g(t)$ on the interval $[0, 8]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema. For each answer blank, write NONE if appropriate.

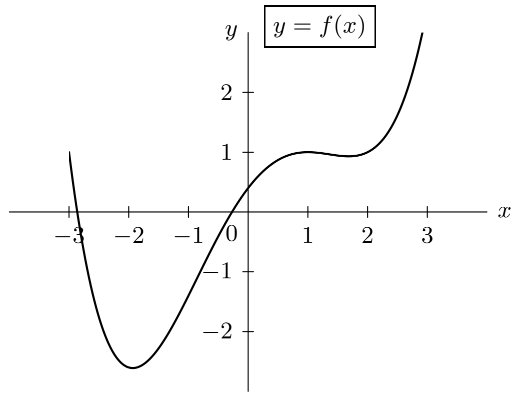
Answer: Global max(es) at $t =$ ______________________

Answer: Global min(s) at $t =$ ______________________

b. [2 points] What is the largest population of bees that occurs in the first 8 months the farm is open?

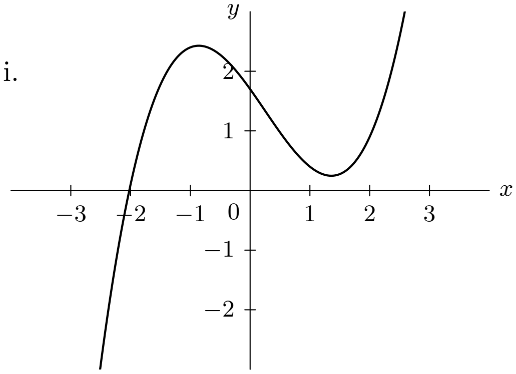
Answer: ______________________

1. [5 points] Below is the graph of a function $f(x)$.

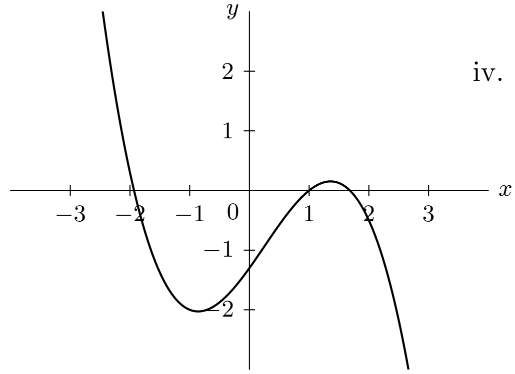


There are six graphs shown below. Circle the <u>one</u> graph that could be the graph of the derivative $f'(x)$.

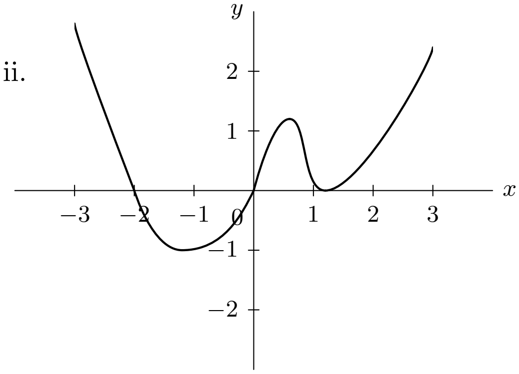
i.



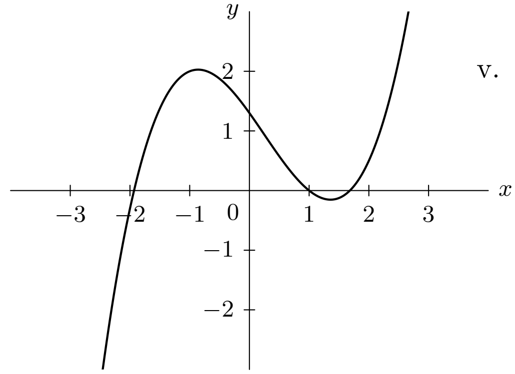
iv.



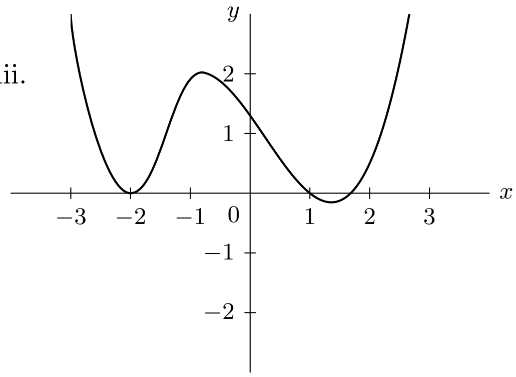
ii.



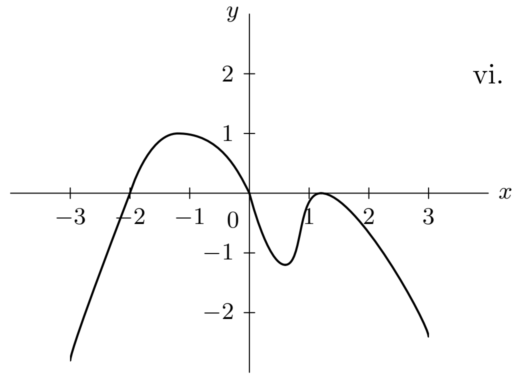
v.



iii.



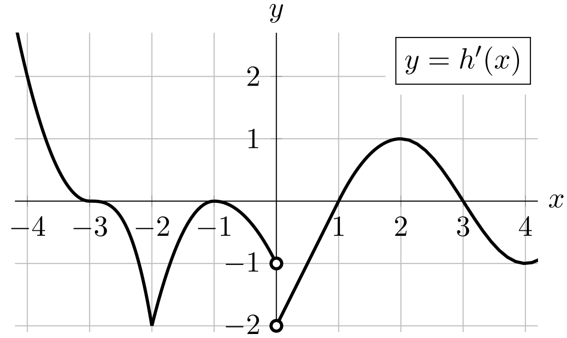
vi.



5. [15 points]
Shown on the right is the graph of $h'(x)$, the **derivative** of a function $h(x)$. Assume that h is continuous on its entire domain $(-\infty, \infty)$.

Use this graph to answer the questions below.

You may also use the fact that $\boxed{h(-4) = 5}$.



a. [3 points] Find the linear approximation $L(x)$ of $h(x)$ near $x = -4$, and use your formula to approximate $h(-3.9)$.

Answer: $L(x) =$ ______________ and $h(-3.9) \approx$ ______________

b. [2 points] Is the estimate of $h(-3.9)$ in part **a.** an overestimate or underestimate of the actual value, or is there not enough information to decide? Briefly explain your reasoning.

Circle one: OVERESTIMATE UNDERESTIMATE NOT ENOUGH INFORMATION

Brief explanation:

For each question below, circle **all** correct choices. You do not need to justify your answers.

c. [2 points] At which of the following values of x does $h(x)$ have a critical point?

$x = -2$ $x = -1$ $x = 0$ $x = 2$ $x = 3$ NONE OF THESE

d. [2 points] At which of the following values of x does $h(x)$ have a local maximum?

$x = -1$ $x = 0$ $x = 1$ $x = 2$ $x = 3$ NONE OF THESE

e. [2 points] At which of the following values of x does $h(x)$ have an inflection point?

$x = -3$ $x = -2$ $x = -1$ $x = 0$ $x = 2$ NONE OF THESE

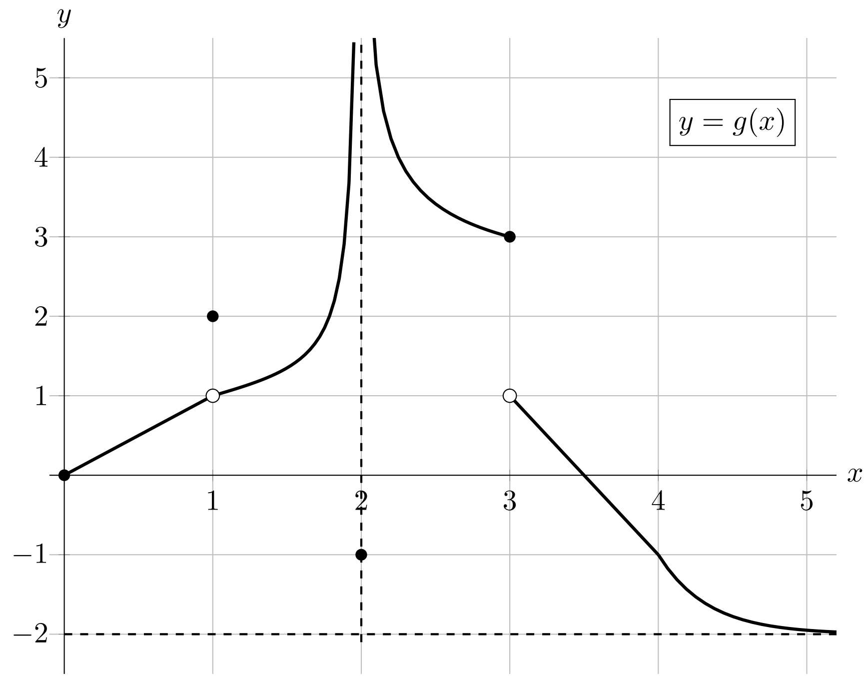
f. [2 points] If $g(x) = h'(x)$, on which of the following interval(s) does $g(x)$ satisfy the hypotheses of the Mean Value Theorem?

$[-4, -1]$ $[-1, 2]$ $[1, 3]$ $[2, 4]$ NONE OF THESE

g. [2 points]. If $g(x) = h'(x)$, on which of the following interval(s) does $g(x)$ satisfy the conclusion of the Mean Value Theorem?

$[-4, -1]$ $[-1, 2]$ $[1, 3]$ $[2, 4]$ NONE OF THESE

1. [9 points] Below is a portion of the graph of an odd function $g(x)$, which has domain $(-\infty, \infty)$ even though the graph below only shows part of the function with $x \geq 0$. Note that $g(x)$ is linear on the intervals $(0,1)$ and $(3,4)$, has a sharp corner at $x = 4$, has a vertical asymptote at $x = 2$, a horizontal asymptote at $y = -2$, and is decreasing for $x > 4$.



a. [1 point] At which of the following values of x is $g(x)$ continuous? *Circle all correct answers.*

$x = 1$ $\qquad$ $x = 2$ $\qquad$ $x = 3$ $\qquad$ $x = 4$ $\qquad$ NONE OF THESE

b. [8 points] Find the **exact** numerical value of each expression below, if possible. For any values that do not exist, including if they are limits that diverge to $\pm\infty$, write DNE. If there is not enough information to find a given value or determine whether it exists, write NEI.
You do not need to show work. As a reminder, $g(x)$ is an odd function.

$g(g(3) - 1) =$ ________ $\qquad$ $\displaystyle\lim_{x \to 3^+} g(x) =$ ________

$\displaystyle\lim_{x \to 4} g(x) =$ ________ $\qquad$ $\displaystyle\lim_{x \to -3^+} g(x) =$ ________

$\displaystyle\lim_{x \to 2} g(x) =$ ________ $\qquad$ $\displaystyle\lim_{h \to 0} \frac{g(3.5 + h) - g(3.5)}{h} =$ ________

$\displaystyle\lim_{x \to 3^-} g(x) =$ ________ $\qquad$ $\displaystyle\lim_{x \to \infty} g(x) =$ ________