

MATH 115 — PRACTICE FOR EXAM 1

Generated October 14, 2026

UMID: SOLUTIONS INITIALS: _____

INSTRUCTOR: _____ SECTION NUMBER: _____

1. This exam has 10 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You may not use a calculator. You are allowed one double-sided 8×11 inch page of handwritten notes.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. You must use the methods learned in this course to solve all problems.
7. You are responsible for reading and following all directions provided for each exam in this course.

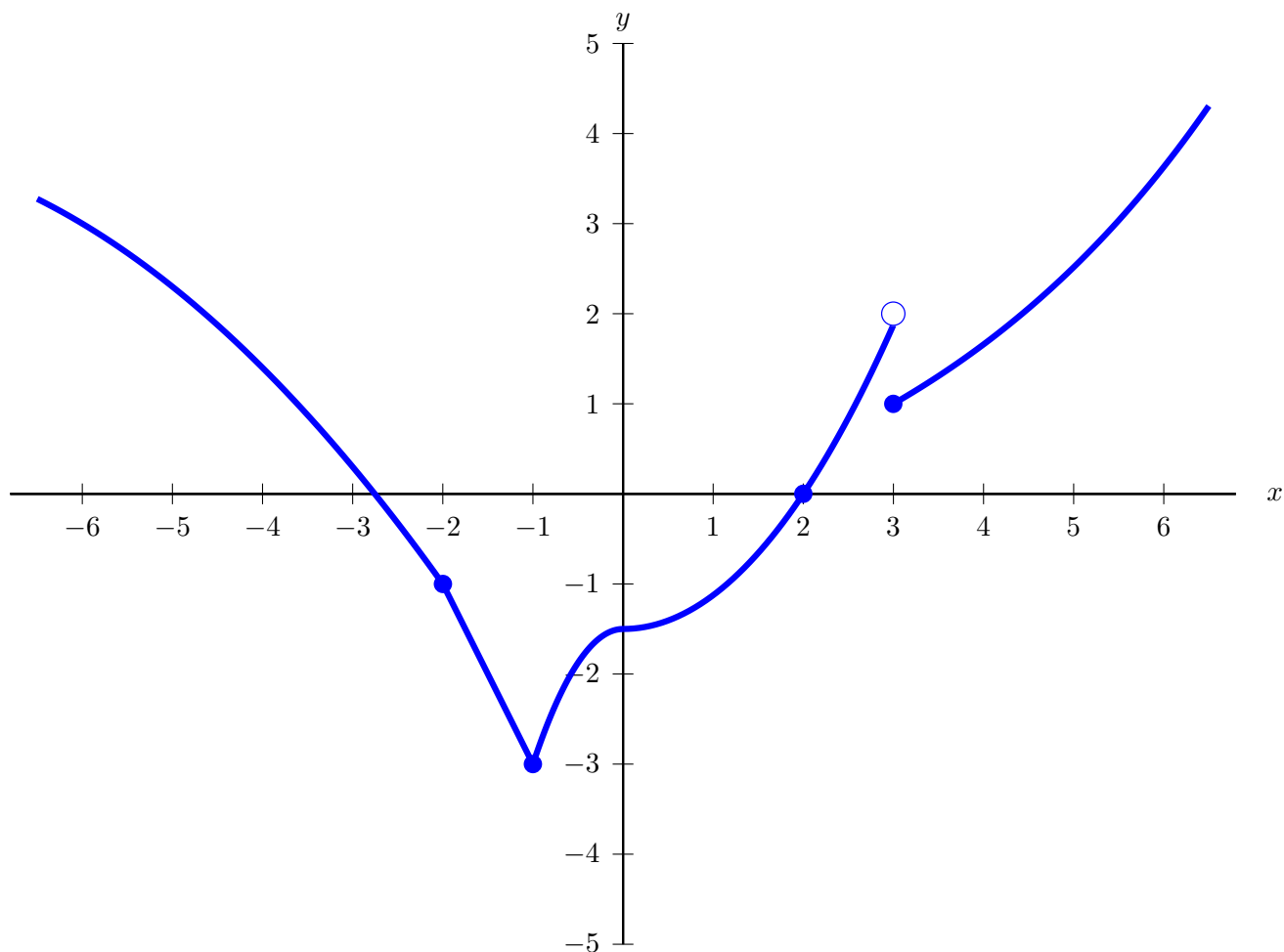
Semester	Exam	Problem	Name	Points	Score
Winter 2015	1	10		10	
Winter 2022	2	8	prairie dogs	8	
Winter 2022	2	3		10	
Winter 2026	1	6		11	
Fall 2017	2	5	Blizzard melting	8	
Fall 2024	2	10	toboggan	10	
Fall 2023	2	8		8	
Fall 2018	1	1	unicycle	11	
Fall 2023	1	3		4	
Fall 2023	1	4		5	
Total				85	

Recommended time (based on points): 90 minutes

10. [10 points] On the axes provided below, sketch the graph of a single function $y = h(x)$ satisfying all of the following:

- $h(x)$ is defined for all x in the interval $-6 < x < 6$.
- $h'(x) < 0$ for all $x < -3$.
- $\lim_{x \rightarrow -2^+} h(x) = -1$.
- $h'(0) = 0$.
- The average rate of change of $h(x)$ between $x = -1$ and $x = 2$ is 1.
- $h(x)$ is not continuous at $x = 3$.
- $h(x) > 0$ for all $x > 3$.
- $h'(x) > 0$ for all $x > 4$.

Make sure that your sketch is large and unambiguous.



8. [8 points] Prairie dogs named Paws and Dot have been hard at work digging a tunnel.

Consider the functions L and C defined as follows:

- $L(w)$ is the length of the tunnel, in feet, when w pounds of dirt have been removed.
- $C(w)$ is the total number of Calories the prairie dogs have burned digging their tunnel when they have removed a total of w pounds of dirt for their tunnel.

The functions $L(w)$ and $C(w)$ are both invertible and differentiable.

- a. [4 points] Complete the sentence below to give a practical interpretation of the equation

$$(L^{-1})'(10) = 24.$$

In order to increase the length of the tunnel from 10 feet to 10.25 feet, ...

Solution: the prairie dogs have to remove approximately 6 pounds of dirt.

- b. [4 points]

- i. Which of the following expressions gives the length, in feet, of the prairie dog tunnel when the prairie dogs have burned a total of x Calories digging? Circle the one correct expression.

$$C(L^{-1}(x))$$

$$C^{-1}(L(x))$$

$$L(C^{-1}(x))$$

$$L^{-1}(C(x))$$

- ii. Use the answer you selected in part i to find an expression for the instantaneous rate of change of the length of the prairie dog tunnel, in feet per calorie, when the prairie dogs have burned a total of 2000 calories digging.

Simplify as much as possible. Note that your final answer may involve the function names

L , L^{-1} , L' , C , C^{-1} , and C' but should not involve the function names $(L^{-1})'$ or $(C^{-1})'$.

Solution: We are trying to find a simplified expression for $\frac{d}{dx}(L(C^{-1}(x)))$ at $x = 2000$. Using the chain rule and the formula for the derivative of the inverse of a function, we have

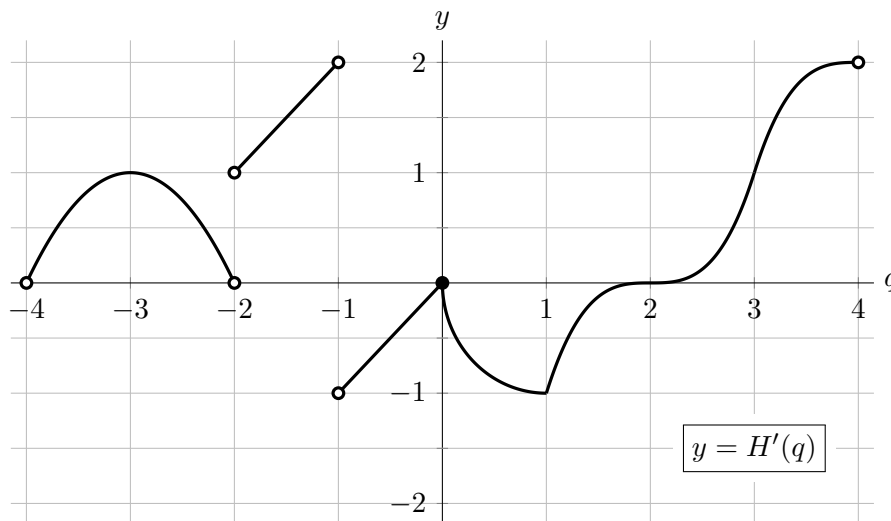
$$\begin{aligned}\frac{d}{dx}(L(C^{-1}(x))) &= L'(C^{-1}(x)) \cdot (C^{-1})'(x) \\ &= L'(C^{-1}(x)) \cdot \frac{1}{C'(C^{-1}(x))}.\end{aligned}$$

So at $x = 2000$, we have

$$L'(C^{-1}(2000)) \cdot \frac{1}{C'(C^{-1}(2000))} = \frac{L'(C^{-1}(2000))}{C'(C^{-1}(2000))}.$$

Answer:
$$L'(C^{-1}(2000)) \cdot \frac{1}{C'(C^{-1}(2000))} = \frac{L'(C^{-1}(2000))}{C'(C^{-1}(2000))}$$

6. [11 points] Suppose $H(q)$ is a **continuous** function defined everywhere on the interval $(-4, 4)$. A graph of the **derivative** $H'(q)$ is given below.



- a. [2 points] Circle all intervals listed below on which $H(q)$ is **increasing**.

☒ $(-4, -3)$ ☒ $(-3, -2)$ ☐ $(0, 1)$ ☐ $(1, 2)$ ☒ $(2, 3)$ NONE OF THESE

- b. [2 points] Circle all intervals listed below on which $H(q)$ is **concave down**.

☐ $(-4, -3)$ ☒ $(-3, -2)$ ☒ $(0, 1)$ ☐ $(1, 2)$ ☐ $(2, 3)$ NONE OF THESE

- c. [2 points] Circle all intervals listed below on which $H''(q)$ is **decreasing**.

☒ $(-4, -3)$ ☒ $(-3, -2)$ ☐ $(0, 1)$ ☒ $(1, 2)$ ☐ $(2, 3)$ NONE OF THESE

- d. [2 points] Circle all points listed below that are **inflection points** of $H(q)$.

☒ $q = -3$ ☒ $q = -2$ ☒ $q = 0$ ☐ $q = 2$ ☐ $q = 3$ NONE OF THESE

- e. [1 point] Circle all points c below for which the limit $\lim_{q \rightarrow c} H(q)$ **does not exist**.

☐ $c = -2$ ☐ $c = -1$ ☐ $c = 0$ ☐ $c = 1$ ☐ $c = 2$ ☒ NONE OF THESE

- f. [1 point] Circle all points c below for which the limit $\lim_{q \rightarrow c} H'(q)$ **does not exist**.

☒ $c = -2$ ☒ $c = -1$ ☐ $c = 0$ ☐ $c = 1$ ☐ $c = 2$ NONE OF THESE

- g. [1 point] Circle all points c below for which the limit $\lim_{q \rightarrow c} H''(q)$ **does not exist**.

☒ $c = -2$ ☐ $c = -1$ ☒ $c = 0$ ☒ $c = 1$ ☐ $c = 2$ NONE OF THESE

5. [8 points] Blizzard the snowman and his mouse friend Gabe arrived in Montana, where it has recently snowed. Since Blizzard is still melting, they decide to use this time to pack extra snow onto Blizzard, to help him make it to the North Pole. Let $H(t)$ be Blizzard's height, in inches, if Blizzard and Gabe stay in Montana for t hours. On the interval $1 \leq t < \infty$, the function $H(t)$ can be modeled by

$$H(t) = 35 + 10e^{-t/6}(t - 2)^{1/3}.$$

Notice that

$$H'(t) = \frac{-5e^{-t/6}(t - 4)}{3(t - 2)^{2/3}}.$$

- a. [6 points] Find all values of t that give global extrema of the function $H(t)$ on the interval $1 \leq t < \infty$. Use calculus to find your answers, and be sure to show enough evidence that the point(s) you find are indeed global extrema. For each answer blank, write NONE if appropriate.

Solution: The critical points of $H(t)$ occur at $t = 4$ and $t = 2$. Also checking endpoints, we have that:

- $H(1) = 26.535$
- $H(2) = 35$
- $H(4) = 41.469$
- $\lim_{t \rightarrow \infty} H(t) = 35$

and so we see that H has a global maximum when $t = 4$ and a global minimum when $t = 1$.

- b. [2 points] Assuming Blizzard stays in Montana for at least 1 hour, what is the tallest height Blizzard can reach? *Remember to include units.*

Solution: Blizzard's tallest height will occur at the global maximum in the interval. Therefore, Blizzard can reach a height of 41.469 inches (when he's been in Montana for 4 hours).

10. [10 points] Tanya is riding her toboggan down the largest sledding hill in Ann Arbor! Assume that the height $H(x)$ of the hill above its base, in **hundreds** of feet, is given as a function of x , the **horizontal** distance from the top of the hill measured in **hundreds** of feet, where

$$H(x) = 1 - \frac{1}{(x-3)^2}, \quad 0 \leq x \leq 2.$$

- a. [3 points] Write a formula for the linear approximation $L(x)$ of $H(x)$ near $x = 1$.

Solution: We have $H(1) = 1 - \frac{1}{(1-3)^2} = \frac{3}{4}$ and

$$H'(1) = 2(1-3)^{-3} = 2\left(-\frac{1}{8}\right) = -\frac{1}{4}.$$

Therefore

$$L(x) = H(1) + H'(1)(x-1) = \frac{3}{4} - \frac{1}{4}(x-1).$$

Answer: $L(x) = \frac{3}{4} - \frac{1}{4}(x-1)$

- b. [1 point] Use your formula from part a. to approximate the height of the hill at a **horizontal** distance of 90 feet from its top.

Solution: Since 90 feet is 0.9 hundred feet, we get the approximation

$$L(0.9) = \frac{3}{4} - \frac{1}{4}(0.9-1) = \frac{3}{4} - \frac{1}{4}(-0.1) = \frac{3}{4} + \frac{1}{40} = \frac{31}{40}.$$

Answer: $\frac{31}{40}$ hundred feet

- c. [3 points] Assume that, as the toboggan goes down the hill, only its midpoint touches the hill. When the midpoint of the toboggan touches the hill at $x = 1$, the highest point of the toboggan is 76 feet above the base of the hill. Find the length of the toboggan in feet.

Solution: When the midpoint of the toboggan touches the hill at $x = 1$, the slope of the line parallel to the toboggan is given by $H'(1) = -\frac{1}{4}$, and the height above the base of the hill of the midpoint of the toboggan is $H(1) = \frac{3}{4}$ hundred feet, or 75 feet. This means the highest point of the toboggan, which will be its back tip, is 1 foot higher than its midpoint, and therefore 2 feet higher than its front tip. So the toboggan itself forms the hypotenuse of a right triangle of height 2 feet and slope $-\frac{1}{4}$. This triangle has width 8 feet, which makes the hypotenuse

$$\sqrt{2^2 + 8^2} = \sqrt{68} = 2\sqrt{17} \text{ feet long.}$$

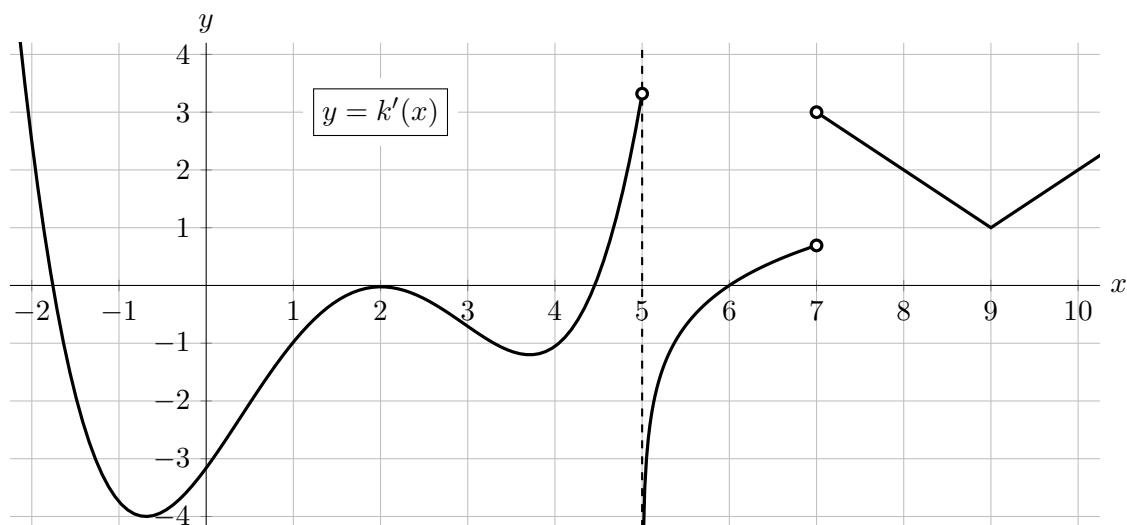
Answer: $2\sqrt{17}$

- d. [3 points] What is the x -value of the point on the hill where the angle between the toboggan and a horizontal line is 45 degrees? Note that $\tan(45^\circ) = 1$.

Solution: The angle between the toboggan and a horizontal line will be 45 degrees wherever the slope of the hill is ± 1 . Since $H(x)$ is decreasing on $0 \leq x \leq 2$, this means we are looking for a point where $H'(x) = -1$. Solving $H'(x) = \frac{2}{(x-3)^3} = -1$, we get $x = 3 - 2^{1/3}$.

Answer: $x = 3 - 2^{1/3}$

8. [8 points] Suppose $k(x)$ is a continuous function, defined for all real numbers. A portion of the graph of $k'(x)$, the **derivative** of $k(x)$, is given below. Note that $k'(x)$ has a vertical asymptote at $x = 5$ and a sharp corner at $x = 9$.



- a. [2 points] Circle the least value that is listed below.

$k(-1)$

$k(0)$

$k(1)$

$k(2)$

$k(3)$

- b. [2 points] Circle the least value that is listed below.

$k''(-2)$

$k''(-1)$

$k''(0)$

$k''(1)$

$k''(2)$

- c. [2 points] Circle all points listed below that are inflection points of $k(x)$.

$x = \frac{1}{2}$

$x = 2$

$x = 3$

$x = 6$

$x = 9$

NONE OF THESE

- d. [1 point] On which of the following intervals does $k'(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle all correct answers.

$[-1, 3]$

$[3, 5]$

$[6, 8]$

$[8, 9]$

$[8, 10]$

NONE OF THESE

- e. [1 point] On which of the following intervals does $k(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle all correct answers.

$[-1, 3]$

$[3, 5]$

$[6, 8]$

$[8, 9]$

$[8, 10]$

NONE OF THESE

1. [11 points] Brianna rides her unicycle north from her home to the grocery store and back again. The differentiable function $r(t)$ represents Brianna's distance in meters from her home t minutes after she leaves the house. Some values of $r(t)$ are shown in the table below.

t	0	1	5	7	10	12	14	16	17
$r(t)$	0	180	1050	1420	1425	980	570	220	0

- a. [2 points] What was Brianna's average velocity between times $t = 7$ and $t = 12$? Include units.

Solution: Average velocity = $\frac{980 - 1420}{12 - 7} = \frac{-440}{5} = -88$ **Answer:** -88 meters per minute.

- b. [2 points] Approximate the value of $r'(14)$. Include units.

Solution: $r'(14) \approx \frac{220 - 570}{2} = -175$ **Answer:** -175 meters per minute.

- c. [3 points] For which of the following time interval(s) is it possible for $r(t)$ to be concave up on the entire interval? Circle all correct choices.

Solution: Computing average rate of changes in consecutive subintervals we see that

Intervals	[1,5]	[5,7]	[10,12]	[12,14]
Average rate of change	$\frac{870}{4} = 217.5$	$\frac{370}{2} = 185$	$-\frac{445}{2} = -222.5$	$-\frac{410}{2} = -205$

Since the average rate of change only increases on $[10, 14]$, then it is possible that $r(t)$ is concave up on **[10, 14]**.

Use the following additional information about Brianna's ride to answer the questions below:

- The grocery store is 1430 meters away from Brianna's home.
 - It takes Brianna 8 minutes to get to the store.
 - On her way to the store, Brianna does not stop at all. On her way back, she only stops once at a traffic light, which is 250 meters from her home.
- d. [2 points] For which of the following time interval(s) is $r'(t)$ equal to 0 for some value of t in that interval? Circle all correct choices.

Solution: Based on the information given $r'(t) \neq 0$ on $[1, 5]$ and $[10, 12]$. $r'(8) = 0$ since it takes 8 minutes to get to the store. Since she stops on her way back, then $r'(t) = 0$ for $14 \leq t \leq 16$.

[1,5]

[5,10]

[10,12]

[12,16]

NONE OF THESE

- e. [2 points] For which of the following time interval(s) is $r'(t)$ negative for some value of t in that interval? Circle all correct choices.

Solution: The derivative of $r(t)$ is negative on her way back.

[1,5]

[5,10]

[10,12]

[12,16]

NONE OF THESE

3. [4 points] Find positive constants a , b , and c such that the function

$$f(x) = \begin{cases} \ln(ce - x) & x \leq 0 \\ \frac{ax^3 + \pi}{x^b + 1} & x > 0 \end{cases}$$

is continuous and satisfies $\lim_{x \rightarrow \infty} f(x) = 4$. Show your work, and write your answers in exact form.

Solution: If $b > 3$ then $\lim_{x \rightarrow \infty} f(x) = 0$, and if $b < 3$ then $\lim_{x \rightarrow \infty} f(x) = \infty$; therefore, since $\lim_{x \rightarrow \infty} f(x) = 4$, we know $b = 3$. Then, since $b = 3$, we know that

$$4 = \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{ax^3 + \pi}{x^3 + 1} = a.$$

Finally, since $f(x)$ is continuous at 0, the two parts of the piecewise definition of $f(x)$ must agree at $x = 0$. Plugging $x = 0$ into both pieces, we get

$$\ln(ce) = \ln(c) + 1 = \pi,$$

so $c = e^{\pi-1}$.

Answers: $a =$ 4 $b =$ 3 $c =$ $e^{\pi-1}$

4. [5 points] Let

$$Q(w) = w^w + \cos(6w - 1).$$

Use the limit definition of the derivative to write an explicit expression for $Q'(3)$. *Your answer should not involve the letter Q . Do not attempt to evaluate or simplify the limit.* Write your final answer in the answer box provided below.

Solution:

$$\begin{aligned} Q'(3) &= \lim_{h \rightarrow 0} \frac{Q(3+h) - Q(3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3+h)^{3+h} + \cos(6(3+h) - 1) - [3^3 + \cos(6 \cdot 3 - 1)]}{h} \end{aligned}$$

Answer: $Q'(3) =$ $\lim_{h \rightarrow 0} \frac{(3+h)^{3+h} + \cos(6(3+h) - 1) - [3^3 + \cos(6 \cdot 3 - 1)]}{h}$

3. [4 points] Find positive constants a , b , and c such that the function

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$$\begin{aligned} Q'(3) &= \lim_{h \rightarrow 0} \frac{Q(3+h) - Q(3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(3+h)^{3+h} + \cos(6(3+h) - 1) - [3^3 + \cos(6 \cdot 3 - 1)]}{h} \end{aligned}$$

Answer: $Q'(3) =$ $\lim_{h \rightarrow 0} \frac{(3+h)^{3+h} + \cos(6(3+h) - 1) - [3^3 + \cos(6 \cdot 3 - 1)]}{h}$