Math 116 — Practice for Exam 1

Generated February 2, 2026

UMID: ____ Initials: ______

Instructor: ______________________________ Section Number: ____________

1. This exam has 8 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
3. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
4. You are allowed notes written on two sides of a $3'' \times 5''$ note card. You are NOT allowed other resources, including, but not limited to, notes, calculators or other electronic devices.
5. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
6. Problems may ask for answers in exact form. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2021	1	3	race	13	
Fall 2011	1	7	AC current	13	
Winter 2024	1	1		16	
Winter 2021	1	7		6	
Fall 2022	1	2		15	
Winter 2018	1	8		14	
Winter 2015	1	2	dog bowl	13	
Winter 2023	1	8	asteroid	12	
Total				102	

Recommended time (based on points): 92 minutes

3. [13 points] Miley and Kylie see their friend Brian in the distance and decide to race to see who can reach him first. However, they see Brian begin pacing back and forth so depending on when they start the race, they will run a different amount. The distance they run, in meters, if the race starts t seconds after Brian begins pacing is

$$L(t) = 25 + 4 \int_{-\left(\frac{\pi}{2}\right)^{\frac{1}{3}}}^{t^3} \cos(r^3) r^2 dr$$

Throughout this problem, please give answers in **exact form** and include **units.**

a. [4 points] If Miley and Kylie start the race immediately as Brian begins pacing, what distance will they run? Evaluate any integrals in your answer and remember to include units.

b. [6 points] Miley and Kylie decide they will start the race at the smallest strictly positive time t (i.e. smallest t with $t > 0$) such that $L'(t) = 0$. Find the time at which they will start the race. Make sure to include units.

c. [3 points] Miley and Kylie want to be able to compute $L(t)$ quickly, so they would like $L(t)$ rewritten in the form below. Write $L(t)$ in the form given below with appropriate expressions in place of the blanks.

$$L(t) = \underline{\hspace{6cm}} + \int_0^t \underline{\hspace{6cm}} \, dr$$

7. [13 points] Household electricity in the United States is supplied in the form of an alternating current that varies sinusoidally with a frequency of 60 cycles per second (Hz). The voltage is given by the equation

$$E(t) = 170\sin(120\pi t),$$

where t is given in seconds and E is in volts.

a. [7 points] Using integration by parts, find $\int \sin^2\theta d\theta$. Show all work to receive full credit. (Hint: $\sin^2\theta + \cos^2\theta = 1$.)

b. [6 points] Voltmeters read the root-mean-square (RMS) voltage, which is defined to be the square root of the average value of $[E(t)]^2$ over one cycle. Find the exact RMS voltage of household current.

1. [16 points] Let $f(x)$ be a function that is **even** and **twice differentiable.** Some values of $f(x)$ and $f'(x)$ are given in the table below:

x	0	1	2	3	4
$f(x)$	-3	2	-1	0	5
$f'(x)$	0	4	$\sqrt{2}$	1	e

Use the table above to compute the **exact value** of the following integrals. If there is not enough information to determine the exact value of an integral, write "NEI." You need to evaluate all integrals completely, and your answers should not involve the letter f, but you do not need to simplify your final answers. Show all your work.

a. [3 points] $\int_{-2}^{2} f'(x)\,dx$

Answer: ____________________

b. [4 points] $\int_{1}^{e^2} \frac{f'(\ln(t))}{t}\,dt$

Answer: ____________________

c. [4 points] $\int_{1}^{3} (2w+1)f'(w)\,dw$

Answer: ____________________

d. [5 points] $\int_{1}^{2} 2x^3 f''(x^2)\,dx$

Answer: ____________________

7. [6 points] Split the function $\dfrac{5x^2 - 7x}{(x-1)^2(x+1)}$ into partial fractions with two or more terms. Do not integrate these terms. Be sure to show all work to obtain your partial fractions.

8. [13 points] Let $f(x)$ be a twice differentiable function with

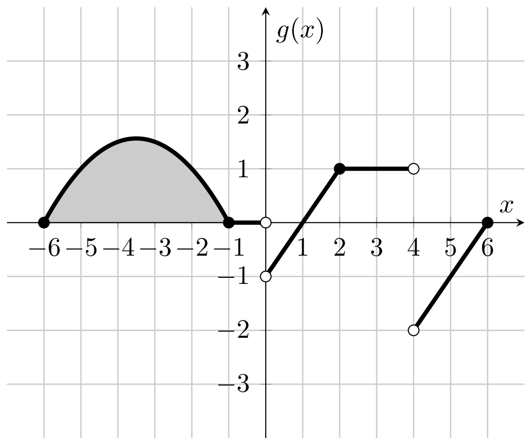
- $f(0) = 1$.
- $f(\ln 2) = \dfrac{5}{4}$.
- $f'(0) = e$.
- $f'(\ln 2) = 2$.

a. [3 points] Compute the average value of $f'(x)$ on $[0, \ln 2]$.

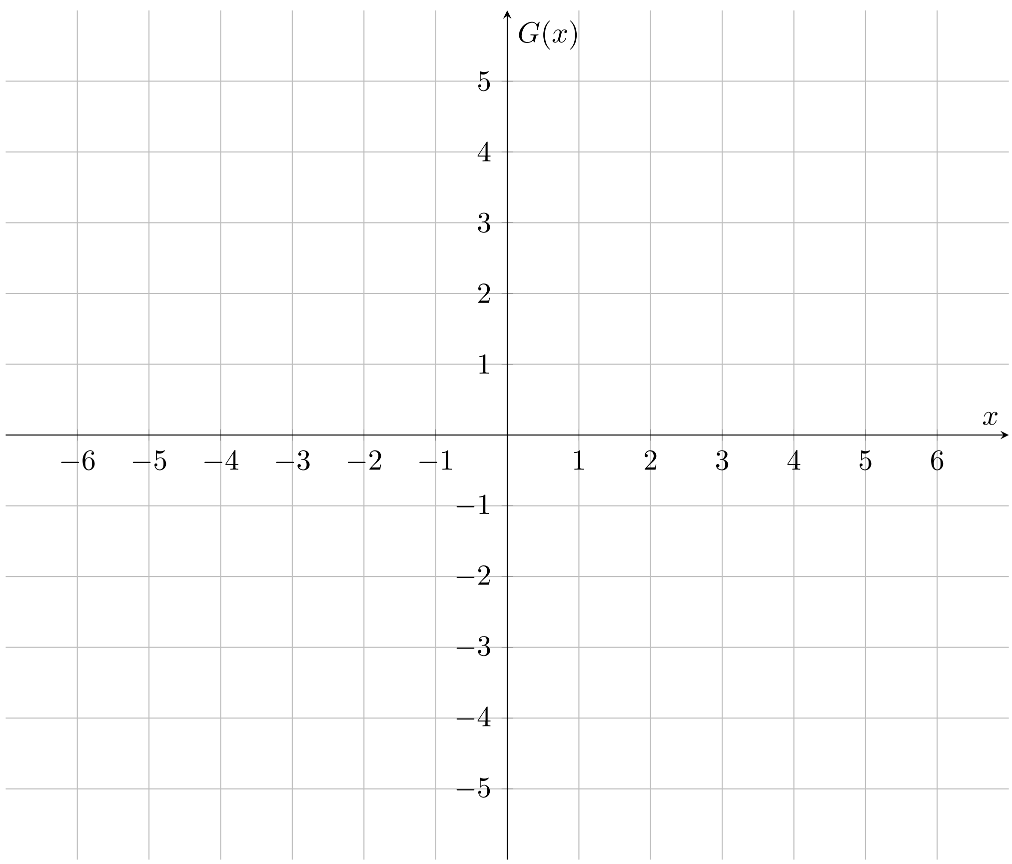
b. [5 points] Compute the exact value of $\displaystyle\int_0^{\ln 2} x f''(x)\, dx$.

c. [5 points] Compute the exact value of $\displaystyle\int_0^{\ln 2} \frac{f'(x)}{\sqrt{9 - (f(x))^2}}\, dx$.

2. [15 points] The function $g(x)$ is graphed below. The area of the shaded region is 5.5. The function $g(x)$ is piecewise linear for $x > -1$.



On the axes provided below, sketch a continuous antiderivative $G(x)$ of $g(x)$ with domain $[-6, 6]$, satisfying $G(1) = 1$. Make sure to clearly label the input and output values at $x = -6, -1, 2, 4,$ and 6. Be sure to make it clear where $G(x)$ is **concave up**, **concave down**, or **linear**, and where it is **increasing**, **decreasing**, or not **differentiable**.



8. [14 points] Let $g(x)$ be a differentiable function with domain $(-1, 10)$ where some values of $g(x)$ and $g'(x)$ are given in the table below. Assume that all local extrema and critical points of $g(x)$ occur at points given in the table.

x	0	1	2	3	4	5	6	7	8
$g(x)$	2.0	3.3	5.7	6.8	6.0	4.3	2.4	0.2	-4.9
$g'(x)$	2.8	2.5	2.0	0.0	-1.4	-1.9	-1.6	-3.0	-8.1

a. [3 points] Estimate $\int_0^8 g(x)\,dx$ using RIGHT(4). Write out each term in your sum.

b. [4 points] Approximate the area of the region between $g(x)$ and the function $f(x) = x + 2$ for $0 \le x \le 4$, using MID(n) to estimate any integrals you use. Use the greatest number of subintervals possible, and write out each term in your sums.

c. [3 points] Is your answer to **b.** an overestimate, an underestimate, or is there not enough information to tell? Briefly justify your answer.

Answer: (circle one)

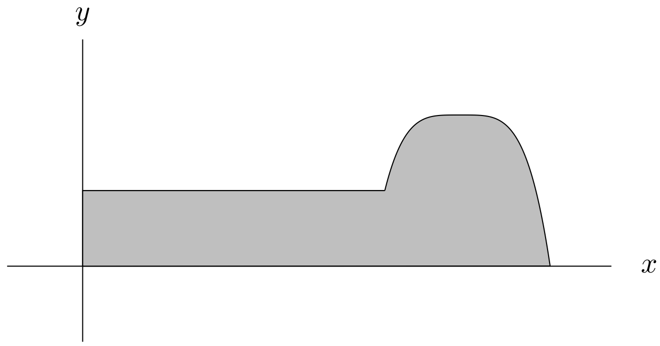
OVERESTIMATE UNDERESTIMATE NOT ENOUGH INFORMATION

d. [4 points] Write an integral giving the arc length of $y = g(x)$ between $x = 2$ and $x = 8$. Estimate this integral using TRAP(2). Write out each term in your sum.

Answer: Integral: ______________________

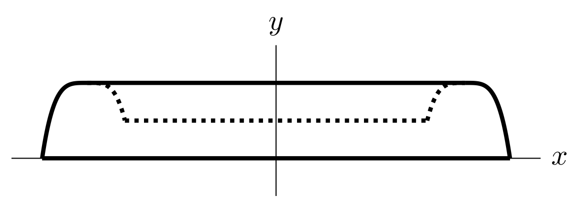
Answer: TRAP(2)= ______________________

2. [13 points] Fred is designing a plastic bowl for his dog, Fido. Fred makes the bowl in the shape of a solid formed by rotating a region in the xy-plane around the y-axis. The region, shaded in the figure below, is bounded by the x-axis, the y-axis, the line $y = 1$ for $0 \leq x \leq 4$, and the curve $y = -(x-5)^4 + 2$ for $4 \leq x \leq 2^{1/4} + 5$. Assume the units of x and y are inches.

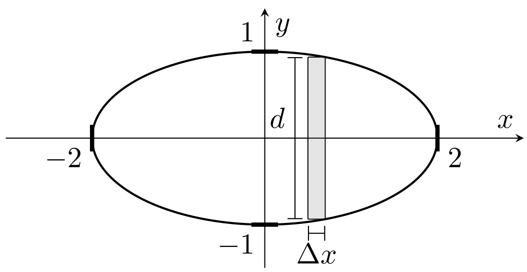


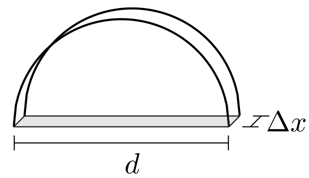
a. [7 points] Write an expression involving one or more integrals which gives the volume of plastic needed to make Fido's bowl. What are the units of your expression?

b. [6 points] Fred wants to wrap a ribbon around the bowl before he gives it to Fido as a gift. The figure below depicts the cross section of the bowl obtained by cutting it in half across its diameter. The thick solid curve is the ribbon running around this cross section, and the dotted curve is the outline of the cross section which is not in contact with the ribbon. Write an expression involving one or more integrals which gives the length of the thick solid curve in the figure (the length of ribbon Fred needs to wrap the bowl).



8. [12 points] Astronomers have spotted a small near-Earth asteroid hurtling towards Earth. In order to assess its danger, they set about calculating its mass. Based on telescope images, the base of the asteroid is given by the region enclosed in the figure on the left, and its cross-sections perpendicular to the x-axis are semi-circles (as shown in the figure on the right). The base is the region bounded by $\frac{x^2}{4} + y^2 = 1$. A sample slice of the base of thickness Δx is shown in graph on the left, and all distances are given in meters.





a. [3 points] Write an expression for the diameter, d, in meters, of a cross-sectional slice of the asteroid x meters from the y-axis.

Answer: $d =$ ______________________

b. [4 points] Write an expression for the volume, V, in m^3, of a cross-sectional slice of the asteroid x meters from the y-axis with thickness Δx meters.

Answer: $V =$ ______________________

c. [2 points] The density of the asteroid depends on x due to shearing (i.e. loss of material) in its direction of travel. The astronomers have computed the expression for the density of a cross-sectional slice of the asteroid to be $\delta(x) = \frac{4000}{7}(x+2)$ kg/m^3. What is the mass, $m(x)$, in kg, of a cross sectional slice of the asteroid with thickness Δx meters?

Answer: $m(x) =$ ______________________

d. [3 points] Write an integral that gives the total mass of the asteroid in kg. Do not evaluate your integral.

Answer: Total Mass = ______________________